Precalculus Unit 6 Homework—Double and Half Angle Formulas

1. Find the exact value of each function.

A. $\tan 105°$

$-2-\sqrt{3}$

B. $\sin\left(\frac{15\pi}{8}\right)$

$-\frac{\sqrt{2-\sqrt{2}}}{2}$

2. Find the exact value of each function if $\cos\theta = \frac{7}{25}$ and $0 < \theta < \frac{\pi}{2}$

a. $\sin 2\theta$

$\frac{336}{625}$

b. $\cos 2\theta$

$\frac{-527}{625}$

c. $\sin\left(\frac{\theta}{2}\right)$

$\frac{3}{5}$

d. $\cos\left(\frac{\theta}{2}\right)$

$\frac{4}{5}$

3. Find the exact value of each function if $\cos\theta = \frac{3}{7}$ and $\frac{3\pi}{2} < \theta < 2\pi$.

a. $\sin 2\theta$

$-\frac{12\sqrt{10}}{49}$

b. $\cos 2\theta$

$\frac{-31}{49}$

c. $\sin\left(\frac{\theta}{2}\right)$

$\frac{\sqrt{14}}{7}$

d. $\cos\left(\frac{\theta}{2}\right)$

$-\frac{\sqrt{35}}{7}$

4. Find the exact value of each function if $\sin\theta = \frac{8}{17}$ and $\frac{\pi}{2} < \theta < \pi$.

a. $\sin 2\theta$

$-\frac{240}{289}$

b. $\cos 2\theta$

$\frac{161}{289}$

c. $\sin\left(\frac{\theta}{2}\right)$

$\frac{4\sqrt{17}}{17}$

d. $\cos\left(\frac{\theta}{2}\right)$

$\frac{\sqrt{17}}{17}$

5. Verify each identify:

A. $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ $\rightarrow \frac{2\cdot\frac{\sin\theta}{\cos\theta}}{\sec^2\theta} = \frac{2\cdot\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}} = 2\frac{\sin\theta}{\cos\theta}\cdot\cos^2\theta = 2\sin\theta\cos\theta$

$= \sin 2\theta$

B. $\sin^2\frac{\theta}{2} = \frac{\tan\theta - \sin\theta}{2\tan\theta}$ $\left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right) = \left(\frac{1-\cos\theta}{2}\right)\left(\frac{\tan\theta}{\tan\theta}\right) = \frac{\tan\theta - \cos\theta\cdot\frac{\sin\theta}{\cos\theta}}{2\tan\theta}$

$= \frac{\tan\theta - \sin\theta}{2\tan\theta}$

C. $\sin 4\theta = 4\cos 2\theta\sin\theta\cos\theta$

$\sin(2\cdot 2\theta) = 2\sin 2\theta\cos 2\theta = 2(2\sin\theta\cos\theta)\cos 2\theta$

$= 4\cos 2\theta\sin\theta\cos\theta$

6. Solve each equation over $[0, 2\pi)$.

A. $\cos(2x) + 3 = 5\cos x$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

B. $\sin 2x\sin x = \cos x$

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

25