Write the partial fraction decomposition of each rational expression.

1) $\dfrac{x-3}{x^2+9x+20}$

$\dfrac{8}{x+5} + \dfrac{-7}{x+4}$

2) $\dfrac{4}{3x^2+x-14}$

$\dfrac{-\frac{12}{13}}{3x+7} + \dfrac{\frac{4}{13}}{x-2}$

3) $\dfrac{-5x+4}{x^2-x}$

$-\dfrac{4}{x} + \dfrac{-1}{x-1}$

4) $\dfrac{2x-2}{(x+5)(x+2)(x-3)}$

$\dfrac{-\frac{1}{2}}{x+5} + \dfrac{\frac{2}{5}}{x+2} + \dfrac{\frac{1}{10}}{x-3}$

5) $\dfrac{-6x^2+3x+5}{x^3-x}$

$-\dfrac{5}{x} + \dfrac{-2}{x+1} + \dfrac{1}{x-1}$

6) $\dfrac{x^2+1}{x^2-3x+2}$ hint: divide first!!!

$1 + \dfrac{5}{x-2} + \dfrac{-2}{x-1}$

7) $\dfrac{2x+4}{x^3-1}$

$\dfrac{2}{x-1} + \dfrac{-2x-2}{x^2+x+1}$

8) $\dfrac{1}{x^3-8}$

$\dfrac{\frac{1}{12}}{x-2} + \dfrac{-\frac{1}{12}x-\frac{1}{3}}{x^2+2x+4}$

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a) $\dfrac{4}{(3x+7)(x-2)} = \dfrac{A}{3x+7} + \dfrac{B}{x-2}$

$4 = A(x-2) + B(3x+7)$

$x=2$: $4 = 13B$, $B = \frac{4}{13}$

$x = -\frac{7}{3}$: $4 = -\frac{13}{3}A$, $A = -\frac{12}{13}$

$$\boxed{\frac{-\frac{12}{13}}{3x+7} + \frac{\frac{4}{13}}{x-2}}$$

4) $\dfrac{2x-2}{(x+5)(x+2)(x-3)} = \dfrac{A}{x+5} + \dfrac{B}{x+2} + \dfrac{C}{x-3}$

$2x-2 = A(x+2)(x-3) + B(x+5)(x-3) + C(x+5)(x+2)$

$x=-2$: $-6 = -15B$, $B = \frac{2}{5}$

$x=3$: $4 = 40C$, $C = \frac{1}{10}$

$x=-5$: $-12 = 24A$, $A = -\frac{1}{2}$

$$\boxed{\frac{-\frac{1}{2}}{x+5} + \frac{\frac{2}{5}}{x+2} + \frac{\frac{1}{10}}{x-3}}$$

6) $x^2-3x+2 \overline{\big)\, x^2+0x+1}$ (quotient: 1)

$-(x^2-3x+2)$

$3x-1$

$1 + \dfrac{3x-1}{x^2-3x+2} \rightarrow 1 + \dfrac{3x-1}{(x-2)(x-1)}$

$\dfrac{3x-1}{(x-2)(x-1)} = \dfrac{A}{x-2} + \dfrac{B}{x-1}$

$3x-1 = A(x-1) + B(x-2)$

$x=1$: $2 = -B$, $B = -2$

$x=2$: $5 = A$

$$\boxed{1 + \frac{5}{x-2} + \frac{-2}{x-1}}$$

e) $\frac{}{x^3-8} = \frac{}{(x-2)(x^2+2x+4)} = \frac{}{x-2} + \frac{}{x^2+2x+4}$

$$1 = A(x^2+2x+4) + (Bx+C)(x-2)$$

$x=2$: $1 = 12A$

$A = \frac{1}{12}$

$$1 = \frac{1}{12}(x^2+2x+4) + (Bx+C)(x-2)$$

$$12 = x^2+2x+4 + 12(Bx+C)(x-2)$$

$$-x^2-2x+8 = 12(Bx+C)(x-2)$$

$$-(x^2+2x-8)$$

$$-(x+4)(x-2) = 12(Bx+C)(x-2)$$

$$-x-4 = 12Bx+12C$$

$-1 = 12B$ $\quad -4 = 12C$

$-\frac{1}{12} = B$ $\quad -\frac{1}{3} = C$

$$\boxed{\frac{\frac{1}{12}}{x-2} + \frac{-\frac{1}{12}x - \frac{1}{3}}{x^2+2x+4}}$$