Precalculus Unit 3 Homework—Practice with Rational Expressions & Graphs

1. Simplify each of the following.

a) $\frac{2a-5}{5-2a}$ -1

b) $\frac{x^3-x}{x+1}$ $x(x-1)$

c) $\frac{2x+1}{4x^2-1}$ $\frac{1}{2x-1}$

d) $\frac{x^2-30x-675}{x^2-6x-1755}$ $\frac{x+15}{x+39}$

e) $\frac{(x+5)-2}{5(x+2)-(x-2)}$ $\frac{1}{4}$

2. Perform the indicated operations and simplify.

a) $\frac{c}{5a}\cdot\frac{15a^2b}{3b^2c}$ $\frac{a}{b}$

b) $\frac{5x-30}{x^2-36}\cdot\frac{3x+18}{5}$ 3

c) $\frac{x^2-3x}{x^2-8x+15}\cdot\frac{x^2-16x+15}{x^2-x}$ $\frac{x-15}{x-5}$

d) $\frac{x^2-9}{x^2-4x-21}\div\frac{4x-12}{3x-21}$ $\frac{3}{4}$

3. Perform the indicated operations and simplify.

a) $\frac{3x}{x-2}-\frac{x+4}{x-2}$ 2

b) $\frac{10}{x-y}-\frac{5}{y-x}$ $\frac{15}{x-y}$

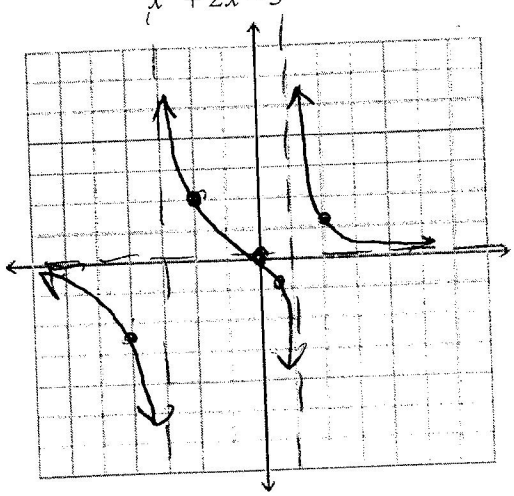
c) $\frac{1}{x-y}-\frac{1}{x+y}$ $\frac{2y}{(x-y)(x+y)}$

d) $\frac{2}{p-5}-\frac{p+11}{p^2-2p-15}$ $\frac{1}{p+3}$

e) $\frac{x^2-5x+78}{18x+x^2-208}-\frac{x}{x+26}$ $\frac{3}{x-8}$

f) $\left[\left(\frac{1}{b}-b\right)\div\left(1-\frac{1}{b}\right)\right](1+b)$ $-1(1+b)^2$

4. For $f(x)=\frac{3x}{x^2+2x-3}$: Find the x- and y- intercepts. Identify any holes and/or asymptotes. Sketch.



no holes

x-int & y-int (0,0)

V.A. $x=1$, $x=-3$

H.A. $y=0$

5. For $f(x)=\frac{x^3+2x^2-2x-2}{x^2+6x+8}$: Identify any holes and/or asymptotes. Describe the end behavior using limits.

oblique asymptote: $y=x-4$

$\lim_{x\to-\infty} f(x)=-\infty$ $\lim_{x\to\infty} f(x)=\infty$

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Practice - Rational Expressions & Graphs

1. a) $\frac{2a-5}{5-2a} = -1$

b) $\frac{x^3-x}{x+1} = \frac{x(x^2-1)}{x+1} = \frac{x\cancel{(x+1)}(x-1)}{\cancel{x+1}} = x(x-1)$

c) $\frac{2x+1}{4x^2-1} = \frac{\cancel{2x+1}}{\cancel{(2x+1)}(2x-1)} = \frac{1}{2x-1}$

d) $\frac{x^2-30x-675}{x^2-6x-1755} = \frac{\cancel{(x-45)}(x+15)}{\cancel{(x-45)}(x+39)} = \frac{x+15}{x+39}$

e) $\frac{(x+5)-2}{5(x+2)-(x-2)} = \frac{x+5-2}{5x+10-x+2} = \frac{x+3}{4x+12} = \frac{x+3}{4(x+3)} = \frac{1}{4}$

2. a) $\frac{\cancel{c}}{\cancel{5a}} \cdot \frac{\cancel{15}a^2\cancel{b}}{\cancel{3}b^{\cancel{2}}\cancel{c}} = \frac{a}{b}$

b) $\frac{5x-30}{x^2-36} \cdot \frac{3x+18}{5} = \frac{\cancel{5}\cancel{(x-6)}}{\cancel{(x+6)}\cancel{(x-6)}} \cdot \frac{3\cancel{(x+6)}}{\cancel{5}} = 3$

c) $\frac{x^2-3x}{x^2-8x+15} \cdot \frac{x^2-16x+15}{x^2-x} = \frac{\cancel{x}\cancel{(x-3)}}{(x-5)\cancel{(x-3)}} \cdot \frac{(x-15)\cancel{(x-1)}}{\cancel{x}\cancel{(x-1)}} = \frac{x-15}{x-5}$

d) $\frac{x^2-9}{x^2-4x-21} \div \frac{4x-12}{3x-21} = \frac{\cancel{(x+3)}\cancel{(x-3)}}{\cancel{(x-7)}\cancel{(x+3)}} \cdot \frac{3\cancel{(x-7)}}{4\cancel{(x-3)}} = \frac{3}{4}$

3. a) $\frac{3x}{x-2} - \frac{x+4}{x-2} = \frac{3x-x-4}{x-2} = \frac{2x-4}{x-2} = \frac{2(x-2)}{x-2} = 2$

c) $\frac{1(x+y)}{(x-y)(x+y)} - \frac{1(x-y)}{(x+y)(x-y)}$ LCD: $(x-y)(x+y)$ $\frac{x+y-x+y}{(x-y)(x+y)} = \frac{2y}{(x-y)(x+y)}$

b) $\frac{10}{-1}\cdot\frac{-1}{(x-y)} - \frac{5}{-(x-y)}$ LCD: $-(x-y)$ $\frac{-10-5}{-(x-y)} = \frac{-15}{-(x-y)} = \frac{15}{x-y}$

e) $\frac{x^2-5x+78}{x^2+18x-208} - \frac{x}{x+26} = \frac{x^2-5x+78}{(x+26)(x-8)} - \frac{x(x-8)}{(x+26)(x-8)}$

LCD: $(x+26)(x-8)$ $\frac{\cancel{x^2}-5x+78-\cancel{x^2}+8x}{(x+26)(x-8)} = \frac{3x+78}{(x+26)(x-8)}$

$\frac{3(x+26)}{(x+26)(x-8)} = \frac{3}{x-8}$

d) $\frac{2(p+3)}{(p+3)(p-5)} - \frac{p+11}{(p-5)(p+3)}$ LCD: $(p+3)(p-5)$ $\frac{2p+6-p-11}{(p+3)(p-5)}$

$\frac{p-5}{(p+3)(p-5)} = \frac{1}{p+3}$

f) $\left[\frac{\frac{1}{b}-b}{1-\frac{1}{b}}\right](1+b) = \left[\frac{\frac{1-b^2}{b}}{\frac{b-1}{b}}\right](1+b) = \left[\frac{(1+b)\cancel{(1-b)}^{-1}}{\cancel{b}}\cdot\frac{\cancel{b}}{\cancel{b-1}}\right](1+b)$

$[-1(1+b)](1+b) = -1(1+b)^2$

4. $f(x) = \frac{3x}{(x-1)(x+3)}$

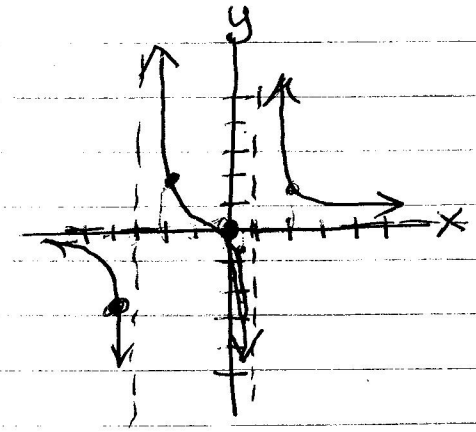
no holes

x-int: $(0,0)$

y-int: $(0,0)$

V.A. $x=1$, $x=-3$

H.A. $y=0$



x	y
-4	$\frac{-12}{5}$
-2	$\frac{-6}{-3}=2$
$\frac{1}{2}$	$\frac{\frac{3}{2}}{-\frac{7}{4}}=-\frac{6}{7}$
2	$\frac{6}{5}$

5. $f(x) = \frac{x^3+2x^2-2x-2}{(x+4)(x+2)}$

LEB: $\lim_{x\to-\infty} f(x) = -\infty$

REB: $\lim_{x\to\infty} f(x) = \infty$

	1	2	-2	2
1	1	3	1	
	1	3	1	-1

oblique asymptote
$y=x-4$
V.A. $x=-4$
$x=-2$

$$\begin{array}{r} x-4 \\ x^2+6x+8 \overline{)\, x^3+2x^2-2x-2} \\ -(x^3+6x^2+8x) \\ \hline -4x^2-10x-2 \\ -(-4x^2-24x-32) \\ \hline 14x+30 \end{array}$$