

Precalculus Unit 8 Test Review

Sequences & Series

1. Determine the 65th term for the sequence: 63, 59, 55, 51... ~~-193~~

2. Determine the common difference for the sequence: $\frac{2}{5}, \frac{16}{15}, \frac{26}{15}, \frac{12}{5}$ $\frac{2}{3}$

3. The ninth term of an arithmetic sequence is 10.6 and the first term is 5. Find the 25th term. 21.8

4. Find the sum of the arithmetic series: $-9 - 5 - 1 + 3 + \dots + 283$ 10138

5. A stack of books is on display in a bookstore. There are 50 books on the first layer, 47 in the second layer, 44 in the third, and so on. There are 17 layers of books.

a. How many books are in the stack? 442

b. How many books are in layers 8 through layer 14 (inclusively)? 140

c. How many books are in row 7? 32

6. Find the 9th term in the sequence 1200, 300, 75, 18.75, ... $\frac{75}{4096}$

7. $\sum_{n=1}^7 243 \left(-\frac{1}{3}\right)^{n-1} = \frac{547}{3}$

8. $\sum_{n=1}^{37} (4n+3) = 2923$

9. $\sum_{n=8}^{48} -4n = -4592$

10. Which term in the geometric sequence 4, 12, 36, ..., is 708588? 12^{+h}

11. Write a recursive formula for the sequence 8, 10, 12, 14, 16, ... $a_1 = 8$
 $a_n = a_{n-1} + 2$

12. Use sigma notation to express the following series. a) $\sum_{n=1}^{10} 0.5(-2)^{n-1}$

a. $0.5 - 1 + 2 - 4 + 8 - 16 + 32 - 64 + 128 - 256$

b. $7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7$ b) $\sum_{n=1}^9 7$

c. $2 + 5 + 8 + 11 + 14 + 17$

d. $20 + 10 + 5 + \dots$

c) $\sum_{n=1}^6 (2+3(n-1))$

d) $\sum_{n=1}^{\infty} 20\left(\frac{1}{2}\right)^{n-1}$

13. A ball is dropped from 200 feet. On each bounce, the ball rises to a height three-fourths of the previous bounce. How far will the ball travel before it stops bouncing? 800 ft

14. Find the sum of the infinite geometric series: $9 + 6 + 4 + \dots$ 27

15. Find the first term in a geometric series where the sum of the first 7 terms is 21589 and the common ratio is 4.

$$\frac{21589}{5461}$$

16. Find the seventh term of the sequence defined by the recursive formula $t_n = 3t_{n-1} + 2$ with $t_1 = 4$.

3644

17. Determine if each sequence is arithmetic, geometric, or neither. Justify your answer.

A. 35, 32, 29, 26, ...

B. 1, 8, 27, 64, 125, ...

C. -1, 6, -36, 216, ...

arithmetic

$d = -3$

neither

pattern is cubic

geometric

$r = -6$

18. Given the explicit formula for the arithmetic sequence, find the first five terms and the 34th term.

$$a_n = -11 + 7n \quad -4, 3, 10, 17, 24$$

$$a_{34} = 227$$

19. Given $a_1 = 28$ and $d = 10$, find the first five terms and the explicit formula for the arithmetic sequence.

$$28, 38, 48, 58, 68 \quad a_n = 28 + 10(n-1)$$

20. Given $a_{38} = -53.2$ and $d = -1.1$, find the first five terms and the explicit formula for the arithmetic sequence.

$$-12.5, -13.6, -14.7, -15.8, -16.9 \quad a_n = -12.5 + -1.1(n-1)$$

21. Given $a_1 = \frac{3}{5}$ and $d = -\frac{1}{3}$, find the recursive formula and the next three terms in the arithmetic sequence.

$$a_1 = \frac{3}{5} \quad a_n = a_{n-1} + -\frac{1}{3} \quad \frac{4}{15}, -\frac{1}{15}, -\frac{2}{5}$$

22. Given $a_{21} = -1.4$ and $d = 0.6$, find the recursive formula and the next three terms in the arithmetic sequence.

$$a_1 = -13.4 \quad a_n = a_{n-1} + 0.6 \quad -0.8, -0.2, 0.4$$

23. Given the explicit formula for the geometric sequence, find the first five terms and the 8th term.

$$a_n = 3^{n-1} \quad 1, 3, 9, 27, 81$$

$$a_8 = 2187$$

24. Given the recursive formula, find the common ratio, the first five terms, and the explicit formula for the geometric sequence.

$$a_n = a_{n-1} \cdot 2 \quad r = 2$$

$$a_1 = 2$$

$$2, 4, 8, 16, 32$$

$$a_n = 2(2)^{n-1}$$

25. Given $a_1 = 0.8$ and $r = -5$, find the first five terms and the explicit formula for the geometric sequence.

$$0.8, -4, 20, -100, 500 \quad a_n = 0.8(-5)^{n-1}$$

26. Given $a_1 = -4$ and $r = 6$, find the recursive formula and the next 3 terms for the geometric sequence.

$$a_1 = -4 \quad a_n = a_{n-1} \cdot 6 \quad -24, -144, -864$$

27. Given $a_4 = 25$ and $r = -5$, find the first five terms, the explicit formula and the recursive formula for the geometric sequence.

$$-\frac{1}{5}, 1, -5, 25, -125 \quad a_n = (-\frac{1}{5})(-5)^{n-1} \quad a_1 = -\frac{1}{5}$$

$$a_n = a_{n-1} \cdot -5$$

28. Given $a_4 = -12$ and $a_5 = -6$, find the 8th term and the recursive formula for the geometric sequence.

$$a_8 = -\frac{3}{4} \quad a_1 = -96$$

$$a_n = a_{n-1} \cdot \frac{1}{2}$$

Partial Fractions

29. Find the partial fraction decomposition: $\frac{5x+7}{x^2+2x-3} = \frac{2}{x+3} + \frac{3}{x-1}$

30. Find the partial fraction decomposition: $\frac{2}{x^3-64} = \frac{\frac{1}{24}}{x-4} + \frac{-\frac{1}{24}x + \frac{1}{3}}{x^2+4x+16}$

Limits (Algebraically)

Evaluate the following limits. If the limit does not exist, give the direction (if it has one).

31. $\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$

32. $\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$

33. $\lim_{x \rightarrow 0} \frac{1}{x} = \text{DNE}$

34. $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$

35. $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$

36. $\lim_{x \rightarrow \infty} \frac{2-6x}{5x+1} = -\frac{6}{5}$

37. $\lim_{x \rightarrow -\infty} \frac{x}{x^3+2} = 0$

38. $\lim_{x \rightarrow \infty} \frac{2x^2+1}{x} = \infty$

$$39. \lim_{x \rightarrow 0} \frac{6x-9}{x^3-12x+3} = -3 \quad 40. \lim_{x \rightarrow 6} \frac{x+6}{x^2-36} = \text{DNE} \quad 41. \lim_{x \rightarrow -2} \frac{x^2-4x+4}{x^2+x-6} = -4$$

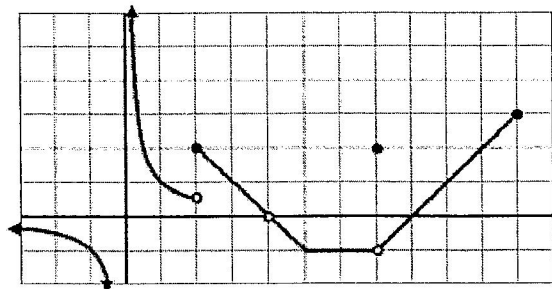
$$42. \lim_{x \rightarrow \infty} 3 = 3 \quad 43. \lim_{x \rightarrow 9} \frac{9-x}{3-\sqrt{x}} = 6 \quad 44. \lim_{x \rightarrow 9} \frac{x^2-9}{x+3} = -6$$

$$45. \lim_{x \rightarrow 4} f(x), f(x) = \begin{cases} \frac{1}{2}x-1, & x \geq 4 \\ 2x-1, & x < 4 \end{cases} \quad \text{DNE} \quad 46. \lim_{x \rightarrow -2} f(x), f(x) = \begin{cases} -x^2+4, & x > -2 \\ 3x+6, & x < -2 \end{cases} = 0$$

Limits (Graphically)

Refer to the graph below to evaluate the following:

47. ∞ $\lim_{x \rightarrow 0^+} f(x)$ 48. $-\infty$ $\lim_{x \rightarrow 0^-} f(x)$ 49. DNE $\lim_{x \rightarrow 0} f(x)$
 50. 2 $\lim_{x \rightarrow 2^+} f(x)$ 51. $\frac{1}{2}$ $\lim_{x \rightarrow 2^-} f(x)$ 52. DNE $\lim_{x \rightarrow 2} f(x)$
 53. -1 $\lim_{x \rightarrow 7^+} f(x)$ 54. -1 $\lim_{x \rightarrow 7^-} f(x)$ 55. -1 $\lim_{x \rightarrow 7} f(x)$
 56. DNE $\lim_{x \rightarrow 11^+} f(x)$ 57. 3 $\lim_{x \rightarrow 11^-} f(x)$ 58. DNE $\lim_{x \rightarrow 11} f(x)$
 59. 0 $\lim_{x \rightarrow 4} f(x)$ 60. -1 $\lim_{x \rightarrow 5} f(x)$ 61. DNE $f(0)$
 62. 2 $f(2)$ 63. DNE $f(4)$ 64. 2 $f(7)$



65. Draw a graph with the following conditions:

♦ $f(0) = 2$

♦ $f(1) = -4$

♦ $f(-4) = 3$

♦ at $f(1)$ there is a non-removable discontinuity

♦ at $f(3)$ there is a removable discontinuity

♦ $\lim_{x \rightarrow -\infty} f(x) = 1$

♦ $\lim_{x \rightarrow \infty} f(x) = -3$

— jump or V.A.

hole

