Proving Trig Identities

1. $\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = \frac{\frac{1}{\sin x}}{\sin x} - \frac{\frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x}} = \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}$

$= \frac{1-\cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$

2. $\frac{\cot x}{\csc^2 x - 1} = \frac{\cot x}{\cot^2 x} = \frac{1}{\cot x} = \tan x$

3. $\tan^2\theta \cos^2\theta + \cot^2\theta \cdot \sin^2\theta = \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta + \frac{\cos^2\theta}{\sin^2\theta} \cdot \sin^2\theta$

$= \sin^2\theta + \cos^2\theta = 1$

4. $\frac{\sec x - \cos x}{\tan x} = \frac{\frac{1}{\cos x} - \cos x}{\frac{\sin x}{\cos x}} = \frac{1-\cos^2 x}{\cos x} \cdot \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x}{\sin x} = \sin x$

5. $\frac{1+\tan x}{\sin x} - \sec x = \frac{1+\frac{\sin x}{\cos x}}{\sin x} - \frac{1}{\cos x} = \frac{\cos x + \sin x - \sin x}{\sin x \cos x}$

$= \frac{\cos x}{\sin x \cos x} = \frac{1}{\sin x} = \csc x$

6. $\sin x \left(\frac{\sin x}{1-\cos x} + \frac{1-\cos x}{\sin x}\right) = \sin x \left(\frac{\sin^2 x + 1 - 2\cos x + \cos^2 x}{(1-\cos x)(\sin x)}\right)$

$= \sin x \left(\frac{2 - 2\cos x}{\sin x (1-\cos x)}\right) = \frac{\sin x \cdot 2(1-\cos x)}{\sin x (1-\cos x)} = 2$

7. $\cos^2\beta - \sin^2\beta = \cos^2\beta - (1-\cos^2\beta) = \cos^2\beta - 1 + \cos^2\beta$
$= 2\cos^2\beta - 1$

8. $\dfrac{\sin^2 t}{\tan^2 t} = \dfrac{\sin^2 t}{\dfrac{\sin^2 t}{\cos^2 t}} = \sin^2 t \cdot \dfrac{\cos^2 t}{\sin^2 t} = \cos^2 t$

9. $\cot^2 y\,(\sec^2 y - 1) = \cot^2 y \cdot \tan^2 y = \dfrac{\cos^2 y}{\sin^2 y} \cdot \dfrac{\sin^2 y}{\cos^2 y} = 1$

10. $\dfrac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right)} = \dfrac{\sin x}{\cos x} = \tan x$

11. $(1+\sin x)(1-\sin x) = 1 - \sin^2 x = \cos^2 x$

12. $(\sin x - \cos x)^2 + 2\sin x - \cos x$
$= \underbrace{\sin^2 x - 2\sin x\cos x + \cos^2 x}_{=1} + 2\sin x - \cos x$
$= 2\sin x - 2\sin x\cos x + 1 - \cos x$ factor by grouping
$= 2\sin x(1-\cos x) + 1\,(1-\cos x)$
$= (1-\cos x)(2\sin x + 1)$