Precalculus Unit 3 Test Review

In exercises 1-2, simplify each rational expression.

1) $\dfrac{x^2+6x+5}{x^2+3x-10} = \dfrac{x+1}{x-2}$

2) $\dfrac{6x^2-7x-3}{8x^2-2x-15} = \dfrac{3x+1}{4x+5}$

In exercises 3-6, perform each of the following operations. Write your answer in simplest form. State any restrictions.

3) $\dfrac{x^2+16x+55}{x^2-8x-65} \bullet \dfrac{x^3-11x^2-26x}{x^2+13x+22} = x$

$x \neq -5, 13, -11, -2$

4) $\dfrac{5x^2}{2x^2+5x-33} \div \dfrac{5x^3-20x}{2x^2+15x+22} = \dfrac{x}{(x-3)(x-2)}$

$x \neq -\frac{11}{2}, 3, 0, -2, 2$

5) $\dfrac{x}{x^2-1} - \dfrac{5}{3x^2-7x-10} = \dfrac{3x^2-15x+5}{(x+1)(x-1)(3x-10)}$

$x \neq -1, 1, 10/3$

6) $\dfrac{\frac{5}{x-2}}{\frac{1}{x-2}+\frac{2}{x+1}} = \dfrac{5(x+1)}{3(x-1)}$

$x \neq 2, -1, 1$

7) Determine the holes, intercepts, asymptotes, and then sketch each of the following:

a) $f(x) = \dfrac{x^2-4}{x^2-9}$

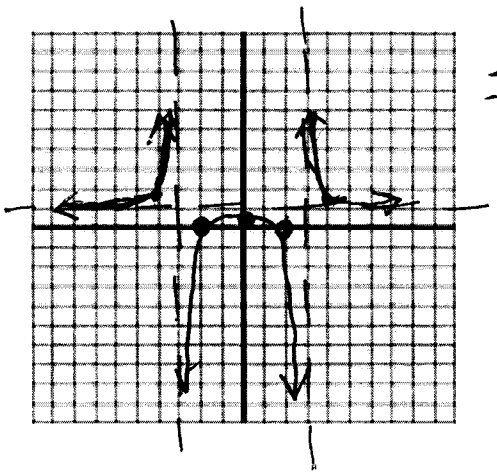
Hole(s): none

x-int: $(-2,0)$ & $(2,0)$

y-int: $(0, \frac{4}{9})$

Eqns of ALL Asymptotes:

V.A. $x=-3, x=3$ H.A. $y=1$



x	y
-4	$\frac{12}{7}$
4	$\frac{12}{7}$

b) $f(x) = \dfrac{3x^2-x-4}{9x^3+9x^2-16x-16}$

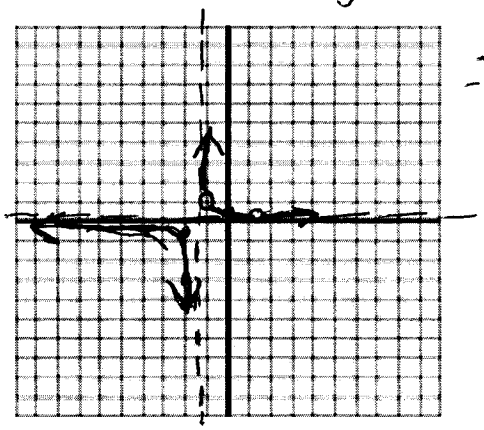
Hole(s): $(-1,1)$ $(\frac{4}{3}, \frac{1}{8})$

x-int: none

y-int: $(0, \frac{1}{4})$

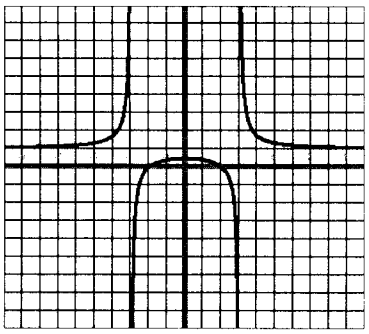
Eqns of ALL Asymptotes:

V.A. $x=-\frac{4}{3}$ H.A. $y=0$



-2	$\frac{10}{-20} = -\frac{1}{2}$

8) Use the graph to the right to fill in the blanks:



$\lim_{x\to-\infty} =$ 1 $\qquad \lim_{x\to-3^-} =$ ∞ $\qquad \lim_{x\to-3^+} =$ $-\infty$

$\lim_{x\to3^-} =$ $-\infty$ $\qquad \lim_{x\to3^+} =$ ∞ $\qquad \lim_{x\to\infty} =$ 1

In exercises 9-10, solve each rational equation. Be sure to check for extraneous solutions.

9) $\dfrac{x-2}{x+4}+\dfrac{x+1}{x+6}=\dfrac{11x+32}{x^2+10x+24}$

$x=5$

10) $\dfrac{3x}{x+5}+\dfrac{1}{x-2}=\dfrac{7}{x^2+3x-10}$

$x=-\frac{1}{3}$

In exercises 11-13, solve each inequality.

11) $2x^2-2x-12>0$

$(-\infty,-2)\cup(3,\infty)$

12) $x^3+9x^2+20x+12<0$

$(-\infty,-6)\cup(-2,-1)$

13) $\dfrac{x^2-4}{x^2+4}\geq 0$

$(-\infty,-2]\cup[2,\infty)$

14) $\dfrac{(x-1)^2}{(x+1)(x+2)}>0$

$(-\infty,-2)\cup(-1,1)\cup(1,\infty)$

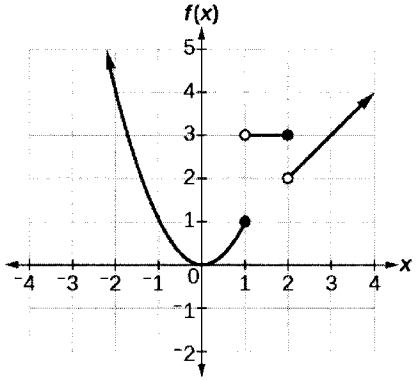
15) $\dfrac{3+x}{3-x}\geq 1$ $\quad [0,3)$

16 $\dfrac{6}{x+3}>x+8$

$(-\infty,-9)\cup(-3,-2)$

REVIEW FROM PREVIOUS UNITS

17) Determine the domain & range and write an equation for the piecewise function.



D: $(-\infty,\infty)$

R: $[0,\infty)$

$$f(x)=\begin{cases} x^2, & x\leq 1 \\ 3, & 1<x\leq 2 \\ x, & x>2 \end{cases}$$

solve $2x^2+9x+19x+15=0$

$x = \frac{3}{2}, \frac{-3 \pm i\sqrt{11}}{2}$

19) Sketch a graph of the polynomial $f(x) = -x(x-2)^4(x+4)^5$. Describe its end behavior, and boundedness.

see work

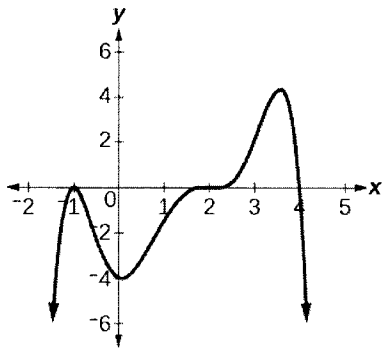
20) Write an equation of the line if $f(2) = 6$ and $f(-4) = 1$.

$y - 6 = \frac{5}{6}(x-2)$

21) Write an equation of the quadratic in vertex form if the vertex is (-1, 4) and it passes through (3, -2).

$y = -\frac{3}{8}(x+1)^2 + 4$

22) Write a possible linear factorization of the graph below:



$y = -(x+1)^2(x-2)^3(x-4)$

23) Describe the graph of $y = 4x^{\frac{2}{3}}$. Determine if it is even, odd, or neither.

see work

24) Determine which statements are true or false for the cubic function $f(x) = (2x-1)(x^2+6x+13)$:

T A) $2x-1$ is a linear factor of $f(x)$

T B) $x-(-3-2i)$ is a factor of $f(x)$

T C) $-3+2i$ is a zero of $f(x)$

F D) the graph of $f(x)$ will cross the x-axis 3 times

T E) there are 2 non-real roots and 1 real root for $f(x)$

25) Determine the two functions that implicitly define the relation $9x^2 - 24xy + 16y^2 = 100$

$y = \frac{3}{4}x - \frac{5}{2}$

$y = \frac{3}{4}x + \frac{5}{2}$

1) $\dfrac{\cancel{(x+5)}(x+1)}{\cancel{(x+5)}(x-2)} = \boxed{\dfrac{x+1}{x-2}}$

2) $\dfrac{(3x+1)\cancel{(2x-3)}}{(4x+5)\cancel{(2x-3)}} = \boxed{\dfrac{3x+1}{4x+5}}$

3) $\dfrac{\cancel{(x+5)}\cancel{(x+11)}}{\cancel{(x+5)}\cancel{(x-13)}} \cdot \dfrac{x\cancel{(x-13)}\cancel{(x+2)}}{\cancel{(x+11)}\cancel{(x+2)}} = \boxed{x}$ $\quad x \neq -5, 13, -11, -2$

4) $\dfrac{\cancel{5}x^{\cancel{2}\,1}}{\cancel{(2x+11)}(x-3)} \cdot \dfrac{\cancel{(2x+11)}\cancel{(x+2)}}{\cancel{5x}\cancel{(x+2)}(x-2)} = \boxed{\dfrac{x}{(x-3)(x-2)}}$ $\quad x \neq \frac{-11}{2}, 0, -2, 2$

5) $\dfrac{x}{(x+1)(x-1)} - \dfrac{5}{(3x-10)(x+1)} = \dfrac{x(3x-10)-5(x-1)}{(x+1)(x-1)(3x-10)}$

$= \dfrac{3x^2-10x-5x+5}{(x+1)(x-1)(3x-10)} = \boxed{\dfrac{3x^2-15x+5}{(x+1)(x-1)(3x-10)}}$ $\quad x \neq -1, 1, \frac{10}{3}$

6) $\dfrac{\frac{5}{x-2}}{\frac{1(x+1)+2(x-2)}{(x-2)(x+1)}} = \dfrac{5}{\cancel{x-2}} \cdot \dfrac{\cancel{(x-2)}(x+1)}{\underbrace{x+1+2x-4}_{3(x-1)}} = \boxed{\dfrac{5(x+1)}{3(x-1)}}$

$x \neq 2, -1, 1$

7) a) $f(x) = \dfrac{(x+2)(x-2)}{(x+3)(x-3)}$

V.A. $x=-3, x=3$

H.A. $y=1$

no holes

x-int: $x=-2$ $(-2,0)$ $(2,0)$

y-int: $(0, \quad)$

b) $f(x) = \dfrac{(3x-4)(x+1)}{(x+1)(3x+4)(3x-4)} = \dfrac{1}{3x+4}$

holes: $\left(-1, \frac{1}{3(-1)+4}\right) \to (-1, 1)$

$\left(\frac{4}{3}, \frac{1}{3(\frac{4}{3})+4}\right) \to \left(\frac{4}{3}, \frac{1}{8}\right)$

V.A. $x = -\frac{4}{3}$

H.A. $y = 0$

x-int: none

y-int: $(0, \frac{1}{4})$

9) LCD $= (x+4)(x+6)$

$(x+6)(x-2) + (x+4)(x+1) = 11x+32$

$x^2+4x-12 + x^2+5x+4 = 11x+32$

$2x^2 - 2x - 40 = 0$

$2(x^2 - x - 20) = 0$

$2(x-5)(x+4) = 0$

$\boxed{x=5}$ $\cancel{x=-4}$ extraneous

10) $3x(x-2) + 1(x+5) = 7$

$3x^2 - 6x + x + 5 = 7$

$3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0$

$\boxed{x = -\frac{1}{3}}$ $\cancel{x=2}$ extraneous

11) $2(x^2 - x - 6) > 0$

$2(x-3)(x+2) > 0$

$x=3$ $x=-2$

$+ \quad | \quad - \quad | \quad +$

$-2 \quad 3$

$(-\infty, -2) \cup (3, \infty)$

12) c: 1, 2, 3, 4, 6, 12

l.c.: 1

possible rat. roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r|rrrr} -1 & 1 & 9 & 20 & 12 \\ & & -1 & -8 & -12 \\ \hline & 1 & 8 & 12 & 0 \end{array}$$

$x^2 + 8x + 12 = 0$

$(x+6)(x+2) = 0$

$x = -6, \; x = -2$

$(x+6)(x+2)(x+1) < 0$

$-$ $+$ $-$ $+$

-6 -2 -1

$(-\infty, -6) \cup (-2, -1)$

13) $\dfrac{(x+2)(x-2)}{x^2+4} \geq 0$

zeros: $x = -2, 2$

not undefined

$+$ $-$ $+$

-2 2

$(-\infty, -2] \cup [2, \infty)$

14) $\dfrac{(x-1)^2}{(x+1)(x+2)} > 0$

zeros: 1

undef.: $-1, -2$

$+$ $-$ $+$ $+$

-2 -1 1

$(-\infty, -2) \cup (-1, 1) \cup (1, \infty)$

15) $\frac{3+x}{3-x} \geq 1$

$\frac{3+x}{3-x} - 1 \geq 0$

$\frac{3+x-(3-x)}{3-x} \geq 0$

$\frac{2x}{3-x} \geq 0$

zero: 0

undef: 3

$- \quad + \quad -$

0 3

$[0,3)$

16) $\frac{6}{x+3} - x - 8 > 0$

$\frac{6-x(x+3)-8(x+3)}{x+3} > 0$

$\frac{6-x^2-3x-8x-24}{x+3} > 0$

$\frac{-x^2-11x-18}{x+3} > 0$

$\frac{-(x^2+11x+18)}{x+3} > 0$

$\frac{-(x+9)(x+2)}{x+3} > 0$

$+ \quad - \quad + \quad -$

$-9 \quad -3 \quad -2$

undef: $x=-3$

zero: $x=-9$, $x=-2$

$(-\infty,-9) \cup (-3,-2)$

18. c: 1, 3, 5, 15

l.c.: 1, 2

possible ratnl. zeros: $\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & 9 & 19 & 15 \\ & & -3 & -9 & -15 \\ \hline & 2 & 6 & 10 & 0 \end{array}$$

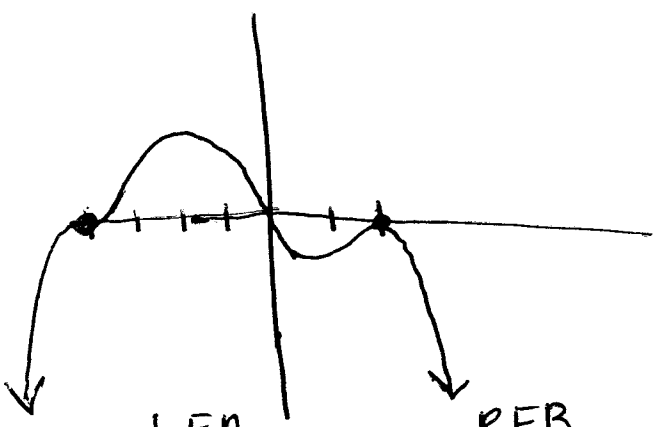
$$\boxed{-\frac{3}{2},\ \frac{-3 \pm i\sqrt{11}}{2}}$$

$2x^2 + 6x + 10 = 0$

$2(x^2 + 3x + 5) = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(5)}}{2(1)} = \frac{-3 \pm \sqrt{-11}}{2} = \frac{-3 \pm i\sqrt{11}}{2}$$

19.



zeros: 0, 2, −4

thru bounce flatten

even degree, neg leadin coeff.

ends: ↙ ↘

bounded above

LEB REB

$$\boxed{\lim_{x \to -\infty} f(x) = -\infty \qquad \lim_{x \to \infty} f(x) = -\infty}$$

20. (2, 6) (−4, 1)

$$m = \frac{1-6}{-4-2} = \frac{-5}{-6} = \frac{5}{6}$$

$$y - 6 = \frac{5}{6}(x - 2)$$

21. $y = a(x-h)^2 + k$

$-2 = a(3+1)^2 + 4$

$-2 = 16a + 4$

$-6 = 16a$

$a = -\frac{3}{8}$ $\qquad y = -\frac{3}{8}(x+1)^2 + 4$

22. $y = -(x+1)^2(x-2)^3(x-4)$

23. $y = 4x^{\frac{2}{3}}$

D: $(-\infty, \infty)$

ends look like root function

y-axis symmetry $\Rightarrow$ EVEN

$f(-x) = 4(-x)^{2/3} = 4(-1)^{2/3}(x)^{2/3}$

$= 4x^{2/3}$

24. A) true

B) true

C) true

D) false – only has 1 real root

E) true

$x^2 + 6x + 13 = 0$

$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(13)}}{2(1)} = \frac{-6 \pm \overset{4i}{\sqrt{-16}}}{2} = -3$

25. $(3x-4y)(3x-4y) = 100$

$(3x-4y)^2 = 100$

$3x - 4y = \pm\sqrt{100} = \pm 10$

$-4y = -3x \pm 10$

$y = \frac{-3x \pm 10}{-4} \Rightarrow y = \frac{3}{4}x - \frac{5}{2}$ and $y = \frac{3}{4}x + \frac{5}{2}$