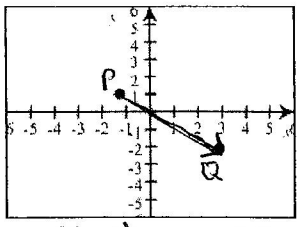
Precalculus Unit 7 Homework—Vectors in the Plane

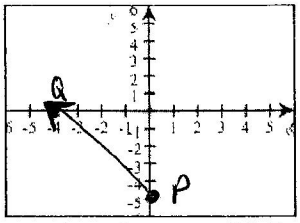
1) Find and draw the vector **v** with initial point P and terminal point Q. Also find the magnitude of **v**.

a) $P(-1,1), Q(3,-2)$



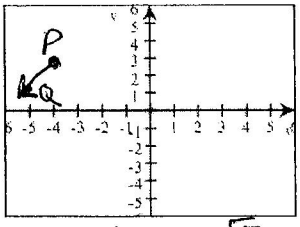
$\mathbf{v} = \langle 4,-3 \rangle$ $|\mathbf{v}| = 5$

b) $P(0,-5), Q(-4,0)$



$\mathbf{v} = \langle -4,5 \rangle$ $|\mathbf{v}| = \sqrt{41}$

c) $P(-4,3), Q(-5,1)$



$\mathbf{v} = \langle -1,-2 \rangle$ $|\mathbf{v}| = \sqrt{5}$

2) Show work to determine if the vector **v** with initial point (p_1, p_2) and terminal point (q_1,q_2) is equivalent to vector **w** with initial point (r_1,r_2) and terminal point (s_1,s_2)

a) **v** (5,3),(−2,2) **w**(7,−1),(0,−2)

$\vec{v} = \langle -7,-1 \rangle$ $\vec{w} = \langle -7,-1 \rangle$

same component form ⇒ equal vectors

b) **v** (−10,−3),(−1,−12) **w**(7,−1),(−2,8)

$\vec{v} = \langle 9,-9 \rangle$ $\vec{w} = \langle 9, 9 \rangle$

not same component form ⇒ not equal

3) Given the vectors $\mathbf{u} = \langle -1,7 \rangle$, $\mathbf{v} = \langle 3,-1 \rangle$, find the following:

a) $\mathbf{u} + \mathbf{v}$

$\langle 2,6 \rangle$

b) $\mathbf{u} - \mathbf{v}$

$\langle -4,8 \rangle$

c) $4\mathbf{u} - 3\mathbf{v}$

$\langle -13,31 \rangle$

d) $\mathbf{u} \cdot \mathbf{v} = -10$

* will learn parts d-g tomorrow!

e) $\text{proj}_{\mathbf{v}}\mathbf{u}$

$\langle -3,1 \rangle$

f) write **u** as the sum of 2 orthogonal vectors (one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$)

$\langle -3,1 \rangle + \langle 2,6 \rangle$

g) the angle between **u** and **v**

116.57°

4) Find a unit vector in the direction of the following vectors and show that it has length 1.

a) $\mathbf{v} = \langle 8,-15 \rangle$

$\left\langle \frac{8}{17}, \frac{-15}{17} \right\rangle$

b) $\mathbf{v} = \langle 3,0 \rangle$

$\langle 1,0 \rangle$

c) $\mathbf{v} = \langle -4\sqrt{2}, -2 \rangle$

$\left\langle \frac{-2\sqrt{2}}{3}, -\frac{1}{3} \right\rangle$

5) Let **u** be the vector with initial point (1, −8) and terminal point (−1, −5) and let $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$. Write the following as a linear combination of **i** and **j**.

$\vec{u} = \langle -2,3 \rangle$

a) $-2\mathbf{u}$

$-2\langle -2,3 \rangle = \langle 4,-6 \rangle$

$4\mathbf{i} - 6\mathbf{j}$

b) $\mathbf{u} - 2\mathbf{v}$

$\langle -2,3 \rangle - 2\langle 3,-4 \rangle$

$\langle -2,3 \rangle + \langle -6,8 \rangle$

$\langle -8,11 \rangle$

$-8\mathbf{i} + 11\mathbf{j}$

c) $\dfrac{\mathbf{u}}{|\mathbf{v}|}$

Write the vector **v** given its magnitude and direction angle.

a) $|\mathbf{v}| = 6 \quad \theta = 45°$ **b)** $|\mathbf{v}| = 12 \quad \theta = 240°$ **c)** $|\mathbf{v}| = 10 \quad \theta =$ direction of $6\mathbf{i} - 2\mathbf{j}$

$\langle 3\sqrt{2}, 3\sqrt{2} \rangle$ $\langle -6, -6\sqrt{3} \rangle$

$\langle 6, -2 \rangle$
magnitude $= \sqrt{40} = 2\sqrt{10}$
$\langle 10 \cdot \frac{6}{2\sqrt{10}}, 10 \cdot \frac{-2}{2\sqrt{10}} \rangle$
$\langle \frac{30}{\sqrt{10}}, -\frac{10}{\sqrt{10}} \rangle$
$\langle 3\sqrt{10}, -\sqrt{10} \rangle$

7) A plane is flying on a bearing of 295° at 360 mph. A wind is blowing with the bearing 320° at 38mph.

a) Write a vector (in component form) of the velocity produced by the airplane alone.

$\mathbf{p} = \langle -326.27, 152.14 \rangle$

b) Write a vector (in component form) of the velocity of the wind.

$\mathbf{w} = \langle -24.43, 29.11 \rangle$

c) Write a vector (in component form) of the actual velocity of the plane.

$\mathbf{v} = \langle -350.70, 181.25 \rangle$

d) Find the actual speed and direction angle (not the bearing) of the plane.

speed $= 394.77$ mph $\theta = 152.67°$

8) A boat is traveling at a bearing of 95° at 25 mph for 2 hours, then it changes direction to a bearing of 135° for 3 hours. What is their bearing and distance from the original starting place?

⟨skip⟩

9) Find the vector projection **u** onto **v.** Then write **u** as a sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$

$\mathbf{u} = \langle -5, -2 \rangle$ & $\mathbf{v} = \langle -11, 3 \rangle$

Skip -- will learn tomorrow

$\text{proj}_{\mathbf{v}}\mathbf{u} = \langle \frac{-539}{130}, \frac{147}{130} \rangle$

$\mathbf{u} = \langle \frac{-539}{130}, \frac{147}{130} \rangle + \langle \frac{-111}{130}, -\frac{407}{130} \rangle$