Pre-Calculus Name: ____________________

Notes 1.4—(Part 1)

Goal #1: Students will be able to find the domain of combinations AND compositions of functions.
Goal #2: Students will be able to decompose a function into two functions (neither of which is the identity function).
Goal #3: Students will be able identify implicitly defined functions within relations.

⊗ When dealing with ***SIMPLE*** combinations of functions (such as $f+g$, $f-g$, $f \bullet g$, and, $f \div g$) the domain of the function consists of all numbers that belong to BOTH the domain of f and the domain of g.

Ex1) Given that $f(x)=\sqrt{2-x}$ & $g(x)=\sqrt{2x+5}$ determine the value of $f+g$, $f-g$, $f \bullet g$, and, $f \div g$ and their domains

$(f+g)(x) = \sqrt{2-x} + \sqrt{2x+5}$

$[-\frac{5}{2}, 2]$

$(f-g)(x) = \sqrt{2-x} - \sqrt{2x+5}$

$[-\frac{5}{2}, 2]$

$(f \bullet g)(x) = \sqrt{2-x} \cdot \sqrt{2x+5} = \sqrt{-2x^2-x-10}$

$[-\frac{5}{2}, 2]$

$(f \div g)(x) = \frac{\sqrt{2-x}}{\sqrt{2x+5}} = \sqrt{\frac{2-x}{2x+5}}$

CAN'T ÷ By 0

$(-\frac{5}{2}, 2]$

Now you try ☺
Perform the indicated operation, simplify your result as much as possible, then determine the domain of the resulting function.

a) If $f(x)=\dfrac{3}{x^2-3}$ and $g(x)=\dfrac{-x^2}{x^2-3}$, find $(f+g)(x)$

$(-\infty, -\sqrt{3}) \cup (-\sqrt{3}, \sqrt{3}) \cup (\sqrt{3}, \infty)$

$\frac{3-x^2}{x^2-3} = \frac{-1(x^2-3)}{x^2-3} = -1$

$\boxed{x^2-3=0 \quad x^2=3 \quad x=\pm\sqrt{3}}$

b) If $f(x)=\sqrt{x-10}$ and $g(x)=\sqrt{x+10}$, find $(fg)(x)$

$[10, \infty)$ $[-10, \infty)$

$\sqrt{x^2-100}$ $[10, \infty)$

c) If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{x+3}$, find $(fg)(x)$

$[0, \infty)$ $[-3, \infty)$

$\sqrt{x^2+3x}$ $[0, \infty)$

d) If $f(x)=\sqrt{x-5}$ and $g(x)=\sqrt{x+2}$, find $(f/g)(x)$

$[5, \infty)$ $[-2, \infty)$

$\sqrt{\frac{x-5}{x+2}}$ $[5, \infty)$

⊗ When dealing with ***COMPOSITIONS*** functions (such as $f \circ g$) the domain of the function consists of all x-values in the domain of g that map to $g(x)$ values in the domain of f.

Ex2.a) Given $f(x)=\sqrt{x-1}$ and $g(x)=x^2+1$ Find the composition function and its domain.

a) $f(g(x)) =$ X

b) $g(f(x)) =$ X

$[1, \infty)$

$(-\infty, \infty)$

$[1, \infty)$

this is important

INVERSE

Ex2.b) Given $f(x)=9-x^2$ and $g(x)=\sqrt{x}$ Find the composition function and its domain.

a) $f(g(x)) = 9-x$

$(-\infty, \infty)$

$[0, \infty)$

b) $g(f(x)) = \sqrt{9-x^2}$

$(9-x^2)^{\frac{1}{2}}$

$\boxed{[0, \infty)}$

$\boxed{[-3, 3]}$

$[0, 3]$

$(-\infty,-3)\cup(-3,\infty)$ $(-\infty,\infty)$

Now you try ☺

2.c) Given $f(x)=x^2-1$ and $g(x)=\sqrt{x}$ Find the composition function and its domain.

a) $f(g(x)) = x-1$ $[0,\infty)$

b) $g(f(x)) = \sqrt{x^2-1}$ $[1,\infty)$!! What!!

$[0,\infty)$ $[(-\infty,\infty)]$

2.d) Given $f(x)=\dfrac{1}{x+3}$ and $g(x)=x^2-3$ Find the composition function and its domain.

a) $f(g(x)) = \dfrac{1}{x^2}$ $(-\infty,-3)\cup(-3,0)\cup(0,\infty)$ New

b) $g(f(x)) = \left(\dfrac{1}{x^2+6x+9}\right)-3$

$(-\infty,-3)\cup(-3,\infty)$

⊗ When ***DECOMPOSING*** functions the purpose is to create two functions (not using the identity function) such that their composition IS the given function. So, when given $h(x)$ the goal is to **DEFINE** $f(x)$ **&** $g(x)$ so that $h(x)=f(g(x))$

Ex3) For each of the following $h(x)$, find the functions f and g such that $h(x) = f(g(x))$.

a) $h(x)=(x+1)^2-3(x+1)+4$

$f(x)=x^2-3x+4$

$g(x)=x+1$

b) $h(x)=\sqrt{x^3+1}$

c) $h(x)=9x^2+6x-2$

Now You Try ☺

d) $h(x)=\sqrt[3]{2x+1}$

$f(x)=\sqrt[3]{x}$

$g(x)=2x+1$

e) $h(x)=\dfrac{x+2}{(x+2)^2+1}$

$f(x)=\dfrac{x}{x^2+1}$

$g(x)=x+2$

f) $h(x)=\dfrac{x+5}{x^2+10x+25}$

$f(x)=\dfrac{x}{x^2}$

$g(x)=x+5$

g) $h(x)=\sqrt{\dfrac{1}{x}}$

$f(x)=\sqrt{x}$

$g(x)=\dfrac{1}{x}$

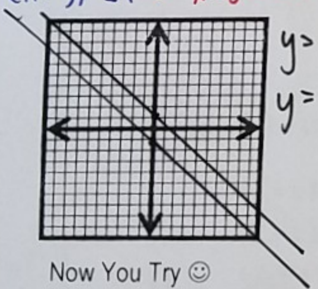
⊗ When dealing with a **RELATION** that is NOT a ***FUNCTION*** it is often possible to solve for y, then identifying the functions which are IMPLICITLY defined by the original relation.

Ex4) Graph of each of the relations below by determining the functions which are implicitly defined within them.

a) $x^2+2xy+y^2=1$

$(x+y)(x+y)=1$

$(x+y)^2=1 \rightarrow x+y=\pm 1$

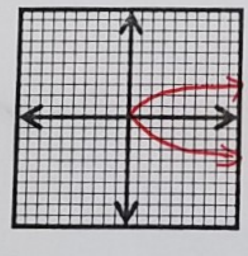


$y=-x+1$

$y=-x-1$

b) $x=y^2$

$y=\pm\sqrt{x}$

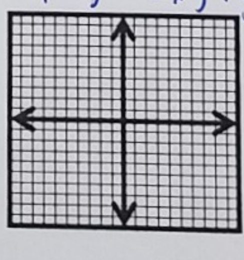


c) $-7x^2+14xy+63=7y^2$

$63=7y^2-14xy+7x^2$

$9=y^2-2xy+x^2$

Solve for y



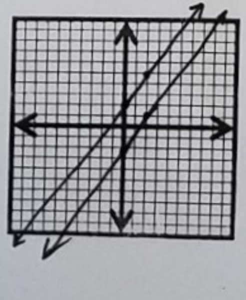
$(x-y)^2=9$

$(x-y)=\pm 3$

$x\pm 3=y$

Now You Try ☺

d) $9x^2-12xy+4y^2=16$



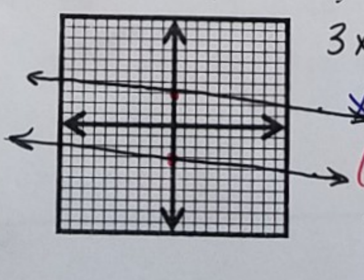
$(3x-2y)^2=16$

$3x-2y=\pm 4$

$3x\pm 4=2y$

$\frac{3}{2}x\pm 2=y$

e) $3x^2+75y^2=27-30xy$



$3x^2+30xy+75y^2=27$

$x^2+10xy+25y^2=9$

$(x+5y)^2=9$

$x+5y=\pm 3$

$5y=-x\pm 3$

$y=-\frac{1}{5}x\pm\frac{3}{5}$