

DEFINITION-----Exponential Functions

An EXPONENTIAL FUNCTION is a function that can be written in the form

The constant a is called the initial value of f
(Notice this is the value of f at $x = 0$)

The constant b is the base

(Notice this is the ONLY number being raised to the x power... a is NOT)

$$f(x) = ab^x$$

- a is non-zero number
- b is a positive number
- $b \neq 1$

*****IDENTIFYING EXPONENTIAL FUNCTIONS*****

Ex1) For each of the following, state whether the function is exponential. If it is, state its initial value, base, and exponent.

(a) $f(x) = 3^x$

yes

$$a = 1$$

$$b = 3$$

$$\text{exponent} = x$$

(b) $g(x) = 6x^{-4}$

no

(c) $h(x) = -2 \cdot 1.5^x$

yes

$$a = -2$$

$$b = 1.5$$

$$\text{exponent} = x$$

(d) $k(x) = 7 \cdot 2^{-x}$

yes

$$7(2^{-1})^x$$

$$a = 7$$

$$b = 2^{-1} = \frac{1}{2}$$

$$\text{exponent} = x$$

(e) $q(x) = 5 \cdot 6^{\pi}$

no

*****EVALUATING EXPONENTIAL FUNCTIONS*****

Ex2) Evaluate each of the following for $f(x) = 2^x$:

$$(a) f(4) = 16 \quad (b) f(0) = 1 \quad (c) f(-3) = \frac{1}{8} \quad (d) f(\frac{1}{2}) = \sqrt{2} \quad (e) f(-3/2) = \frac{1}{2\sqrt{2}}$$

$$2^4 \quad 2^0 \quad 2^{-3} = \frac{1}{2^3} \quad 2^{\frac{1}{2}} \quad 2^{-\frac{3}{2}} = \frac{1}{2^{\frac{3}{2}}} = \frac{1}{2^3} = \frac{1}{\sqrt{8}}$$

*****FINDING EXPONENTIAL FUNCTIONS*****

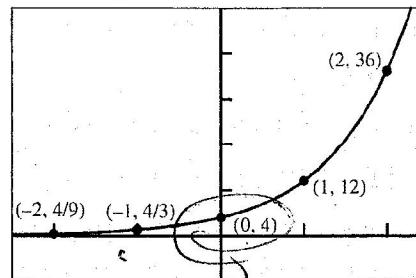
Ex3) Given its table of values or its graph, find the equation of the exponential function:

x	$f(x)$
-2	6/25
-1	6/5
0	6
1	30
2	150

$$* 5 \quad b = 5$$

$$\text{initial value} \quad a = 6$$

(b)



$$* 3 \quad b = 3$$

$[-2.5, 2.5]$ by $[-10, 50]$

$$a = 4$$

growth
 $b > 1$

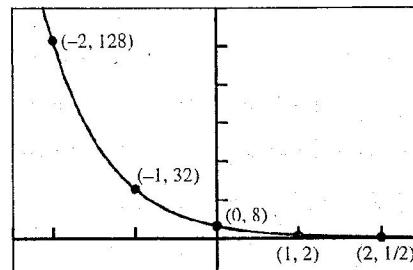
$$f(x) = 6 \cdot 5^x$$

$$f(x) = 4 \cdot 3^x$$

x	$f(x)$
-2	56
-1	28
0	14
1	7
2	7/2

$$a = 14 \\ b = \frac{1}{2}$$

(d)



[-2.5, 2.5] by [-25, 150]

$$a = 8 \\ b = \frac{1}{4}$$

decay

$$0 < b < 1$$

$$f(x) = 8 \cdot \left(\frac{1}{4}\right)^x$$

$$f(x) = 14 \cdot \left(\frac{1}{2}\right)^x$$

***** Transforming Exponential Functions *****

Exponential Functions $f(x) = b^x$

Domain: All reals

Range: $(0, \infty)$

Continuous

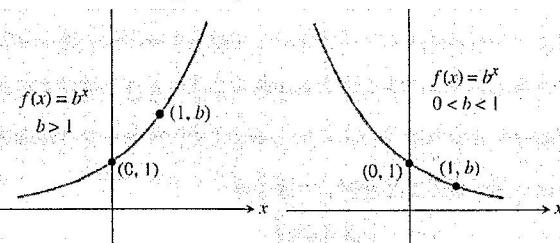
No symmetry: neither even nor odd

Bounded below, but not above

No local extrema

Horizontal asymptote: $y = 0$

No vertical asymptotes



Graphs of $f(x) = b^x$ for (a) $b > 1$ and (b) $0 < b < 1$.

If $b > 1$:

- f is an increasing function,
- $\lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = \infty$.

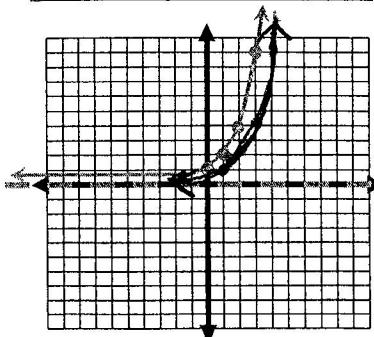
If $0 < b < 1$:

- f is a decreasing function,
- $\lim_{x \rightarrow -\infty} f(x) = \infty$ and $\lim_{x \rightarrow \infty} f(x) = 0$.

Ex4) Describe how to transform the graph of $f(x) = 2^x$ into each of the given functions and sketch.

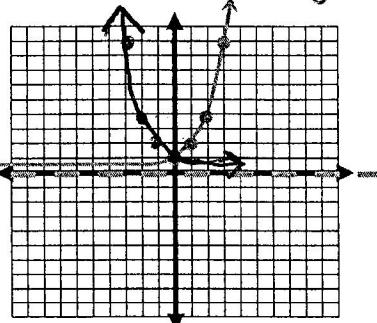
$$g(x) = 2^{x-1}$$

Shift rt. 1



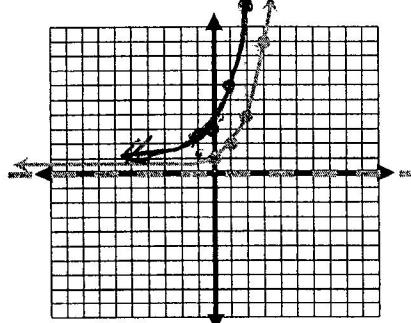
$$h(x) = 2^{-x}$$

reflect over y-axis



$$k(x) = 3 \cdot 2^x$$

vertical stretch *3



***** THE NATURAL BASE e *****

DEFINITION The Natural Base e

$$e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$

$$e \approx 2.718281828459\dots$$

(irrational)

Ex5) Describe how to transform $f(x) = e^x$ into each of the following functions:

$$g(x) = e^{2x} + 2$$

horiz. shrink *2

shift up 2

$$h(x) = -e^{x-3}$$

shift right 3

reflect over x-axis

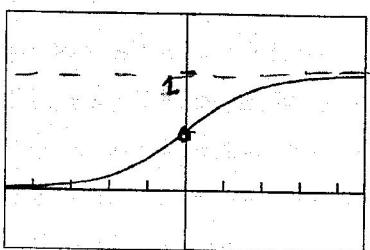
$$k(x) = \frac{1}{2} e^{-x}$$

reflect over y-axis

vert. shrink *1/2

*****LOGISTIC FUNCTIONS*****

BASIC FUNCTION The Logistic Function



[−4.7, 4.7] by [−0.5, 1.5]

The graph of $f(x) = 1/(1 + e^{-x})$.

$$f(x) = \frac{1}{1 + e^{-x}}$$

Domain: All reals

Range: (0, 1)

Continuous

Increasing for all x

Symmetric about $(0, 1/2)$, but neither even nor odd

Bounded below and above

No local extrema

Horizontal asymptotes: $y = 0$ and $y = 1$

No vertical asymptotes

End behavior: $\lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = 1$

DEFINITION-----Logistic Growth Functions

A LOGISTIC GROWTH FUNCTION in x is a function that can be written in the form

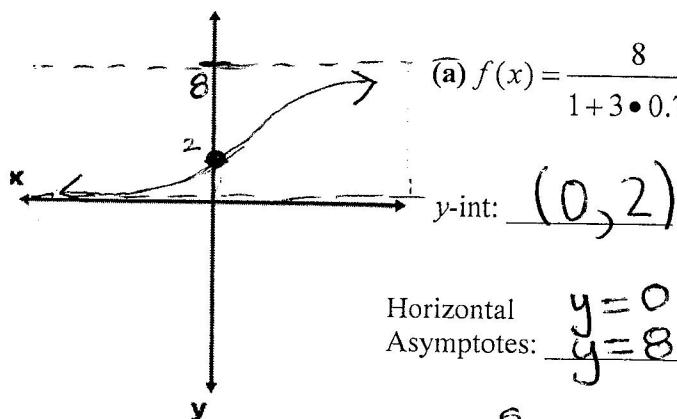
$$f(x) = \frac{c}{1 + a \cdot b^x} \quad \text{or} \quad f(x) = \frac{c}{1 + a \cdot e^{-kx}}$$

where a, b, c , & k are positive constants, $b < 1$ & c is called the limit to growth *Carrying capacity*

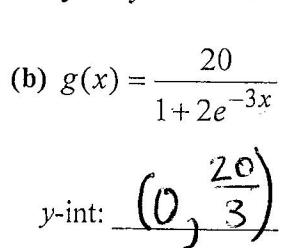
- All logistic growth functions have graphs like the basic logistic function where the end behavior can be described as: $\lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = c$
- All logistic growth functions are bounded by asymptotes $y = 0$ & $y = c$
- All logistic growth functions have a range $(0, c)$

*****GRAPHING LOGISTIC FUNCTIONS*****

Ex6) Sketch each of the following logistic growth functions, identify the y-int & horizontal asymptotes.



$$f(0) = \frac{8}{1 + 3 \cdot 0.7^0} = \frac{8}{1 + 3} = 2$$



$$g(0) = \frac{20}{1 + 2e^{-3(0)}} = \frac{20}{1 + 2 \cdot 1} = \frac{20}{3}$$