Definition: A function is a sinusoid if it can be written in the form → $f(x) = a \cdot \sin(bx + c) + d$
(where a, b, c, and d are constants and neither a nor b is 0) **OR** $f(x) = a \cdot \cos(bx + c) + d$

Definition: The amplitude of a sinusoid of the form → $f(x) = a \cdot \sin(bx + c) + d$ **OR** $f(x) = a \cdot \cos(bx + c) + d$ is $|a|$. Graphically, the amplitude is $\frac{1}{2}$ of the total height.

Definition: The period of a sinusoid of the form → $f(x) = a \cdot \sin(bx + c) + d$ **OR** $f(x) = a \cdot \cos(bx + c) + d$ is $\frac{2\pi}{|b|}$. Graphically, the period is the length of one full cycle.

Definition: The frequency of a sinusoid of the form → $f(x) = a \cdot \sin(bx + c) + d$ **OR** $f(x) = a \cdot \cos(bx + c) + d$ is $\frac{|b|}{2\pi}$. Graphically, the frequency is the number of complete cycles in 2π radians

Things to keep in mind . . .

- The basic graphs of sine and cosine have a period of 2π.
- Changes in amplitude and period as well as phase shifts are nothing more than transformations you've seen before; they have just been given new names for trig functions.
  - Changes in amplitude are vertical stretches or shrinks/compressions
  - Changes in period are horizontal stretches or shrinks/compressions
  - Phase shifts are horizontal (left or right) shifts
  - These graphs can also be shifted vertically

Graphs of Sinusoids

The graphs of $y = a \sin(b(x - h)) + k$ and $y = a \cos(b(x - h)) + k$ (where $a \neq 0$ and $b \neq 0$) have the following characteristics:

amplitude $= |a|$;

period $= \frac{2\pi}{|b|}$;

frequency $= \frac{|b|}{2\pi}$.

When compared to the graphs of $y = a \sin bx$ and $y = a \cos bx$, respectively, they also have the following characteristics:

a phase shift of h;

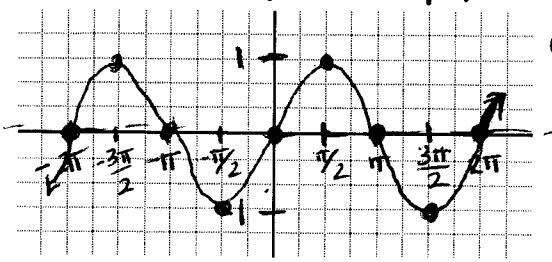
a vertical translation of k.

For the graphs of $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$

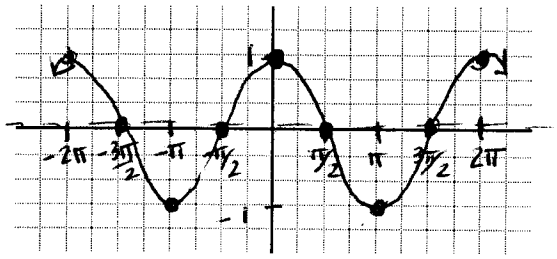
- Amplitude $= |A|$
- Period $= \frac{2\pi}{|B|}$
- Phase Shift $= \frac{C}{B}$
- Vertical Shift $= D$
- Distance Between Key Points* $= \left(\frac{1}{4}\right) \cdot (\text{period})$

*_Key Points_ are the points that are at the top or bottom of the graph, or the points on the center-line of the graph

Graph $y = \sin x$ period $= \frac{2\pi}{|1|} = 2\pi$ $\frac{1}{4}(2\pi) = \frac{\pi}{2}$ amp $= |1| = 1$



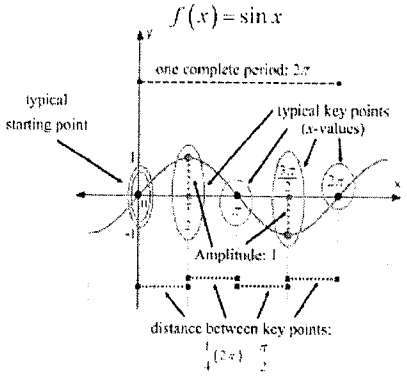
Graph $y = \cos x$



Here are examples of a single change to each of these elements for the basic sine graph.

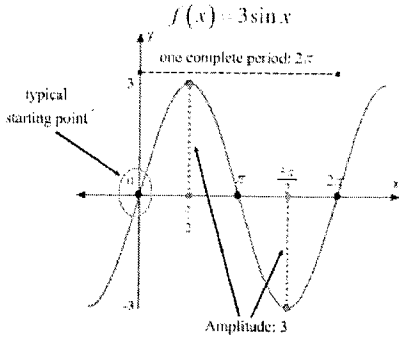
Basic Sine Graph

$f(x) = \sin x$



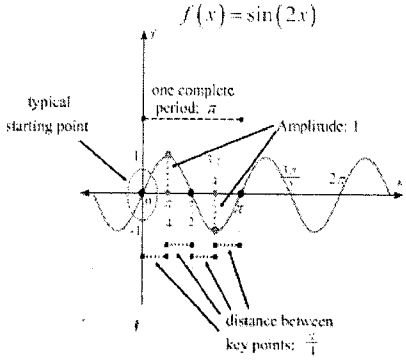
Change in Amplitude

$f(x) = 3\sin x$



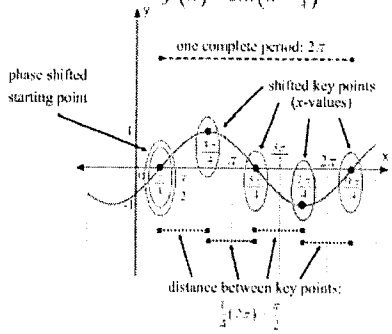
Change in Period

$f(x) = \sin(2x)$



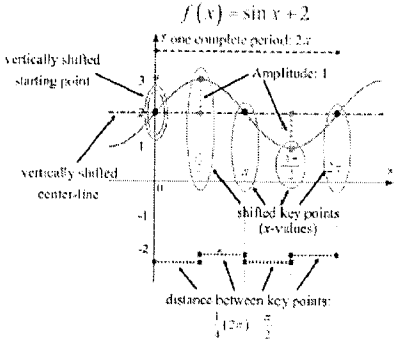
Phase Shift

$f(x) = \sin\left(x - \frac{\pi}{4}\right)$



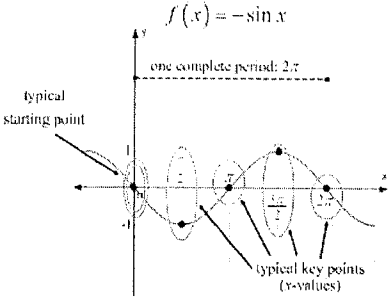
Vertical Shift

$f(x) = \sin x + 2$



Reflection

$f(x) = -\sin x$



Example 1 Find the amplitude of each of the following sinusoids & then use the language of transformations to describe how the graphs of b and c are related to a.

a) $f(x) = \cos x$

amp $= |1| = 1$

b) $y = \frac{1}{2}\cos x$

amp $= \left|\frac{1}{2}\right| = \frac{1}{2}$

vertical shrink $* \frac{1}{2}$

c) $y = -3\cos x$

amp $= |-3| = 3$

vertical stretch $* 3$
reflect over x-axis

Example 2 Find the period of each of the following sinusoids & then use the language of transformations to describe how the graphs of b and c are related to a.

a) $f(x) = \sin x$

pd $= \frac{2\pi}{|1|} = 2\pi$

b) $y = 3\sin(-2x)$

pd $= \frac{2\pi}{|-2|} = \pi$

horiz. shrink $* \frac{1}{2}$
refl. over y-axis
vert. stretch $* 3$

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c) $y = -2\sin\left(\frac{x}{3}\right)$

pd $= \frac{2\pi}{\left|\frac{1}{3}\right|} = 6\pi$

horiz. stretch $* 3$
vert. stretch $* 2$
refl. over x-axis

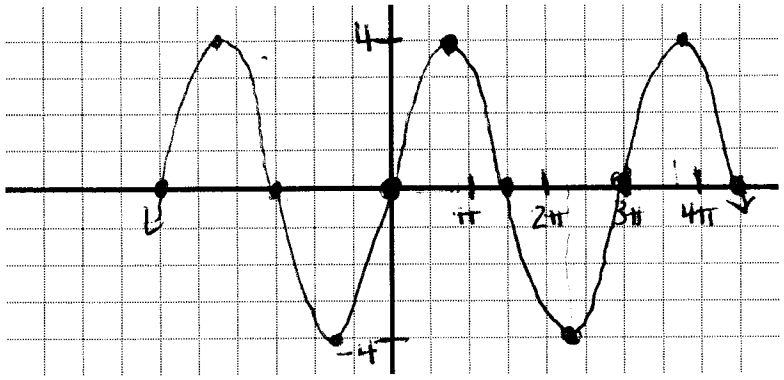
Example 3 Find the amplitude, period, and frequency of the function $f(x) = 4\sin\left(\frac{2x}{3}\right)$. Sketch the graph.

amp $= |4| = 4$

per $= \frac{2\pi}{\left|\frac{2}{3}\right|} = 3\pi$

freq $= \frac{1}{3\pi}$

$\frac{1}{4}(3\pi) = \frac{3\pi}{4}$



Example 4 Find the amplitude, period, phase shift, vertical shift, and any reflection.

$y = -2\cos 4\left(x + \frac{\pi}{4}\right)$

$y = -2\cos(4x + \pi)$

amp $= |-2| = 2$

per $= \frac{2\pi}{|4|} = \frac{\pi}{2}$

p.s. $= \frac{-\pi}{4}$ left $\frac{\pi}{4}$

V.S. none

reflection over x-axis

Example 5 *Find the amplitude, period, phase shift, vertical shift, and any reflection. Then graph one complete period.*

A. $y = 3\sin(6\pi x)$

amp $= |3| = 3$

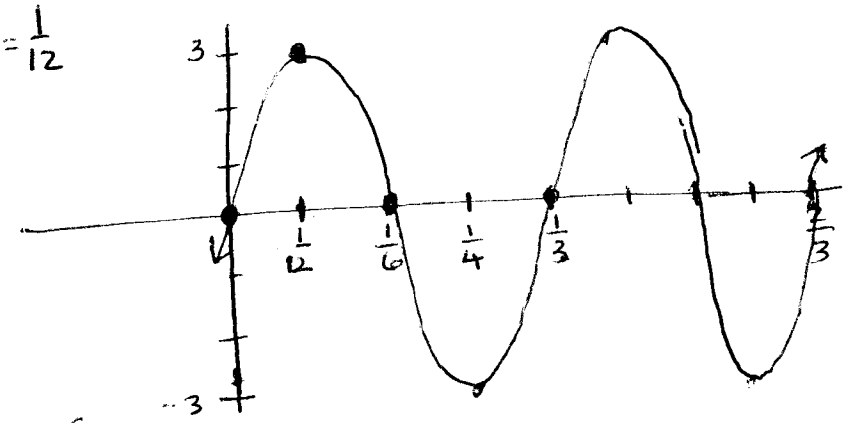
per $= \frac{2\pi}{|6\pi|} = \frac{1}{3}$

p.s. = none

V.S. = none

refl = none

$\frac{1}{4}\left(\frac{1}{3}\right) = \frac{1}{12}$



B. $y = -2\cos(x) + 3$

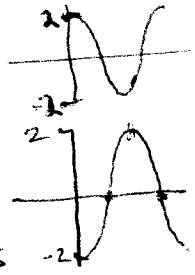
amp $= |-2| = 2$

per $= \frac{2\pi}{|1|} = 2\pi$

p.s. none

V.S. 3 (up 3)

refl. over x-axis



C. $y = -3\sin\left(\frac{\pi}{2}x + \frac{\pi}{4}\right) - 2$

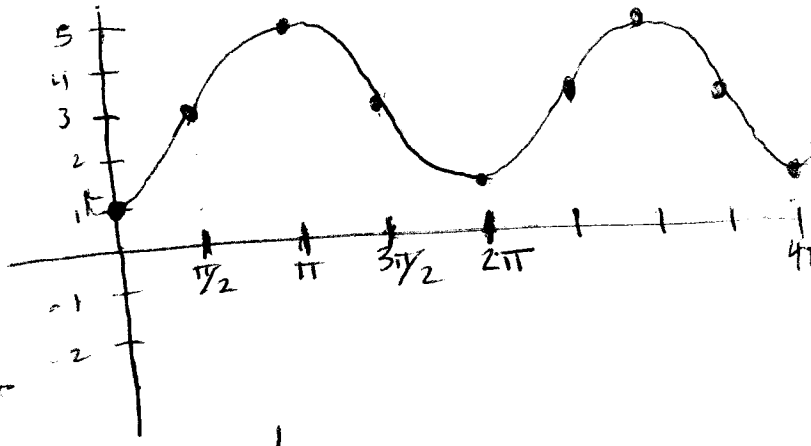
amp $= |-3| = 3$

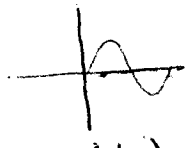
per $= \frac{2\pi}{\left|\frac{\pi}{2}\right|} = 2\pi \cdot \frac{2}{\pi} = 4$

p.s. $\frac{-\frac{\pi}{4}}{\frac{\pi}{2}} = -\frac{\pi}{4} \cdot \frac{2}{\pi} = -\frac{1}{2}$ left $\frac{1}{2}$

V.S. -2 down 2

refl. over x-axis





$\frac{1}{4}(4) = 1$