Integration By Parts

$$\int f(x)\, g'(x) = f(x)\, g(x) - \int f'(x)\, g(x)$$

or

$$\int u\, dv = u \cdot v - \int v\, du$$

$$\int f(x)\,g'(x) = f(x)g(x) - \int f'(x)\,g(x)$$

$$\int e^x \cos x = e^x \sin x - \int e^x \sin x\,dx$$

$f(x) = e^x$
$f'(x) = e^x$

$g(x) = \sin(x)$
$g'(x) = \cos x$

$$\int e^x \cos x = e^x \sin x - \left[-e^x \cos x + \int e^x \cos x\,dx\right]$$

$$= e^x \sin x + e^x \cos - \int e^x \cos x\,dx$$

$$\int e^x \sin x\,dx = -e^x \cos x + \int e^x \cos x\,dx$$

$f(x) = e^x$
$f'(x) = e^x$

$g(x) = -\cos x$
$g'(x) = \sin x$

$$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x\,dx = \frac{e^x \sin x + e^x \cos x}{2}$$

$$\int x \cos x\,dx = f(x)g(x) - \int f'(x)\,g(x)$$

$f(x) = x$
$f'(x) = 1$
$g(x) = \sin x$
$g'(x) = \cos x$

$$= x \sin x - \int \sin dx$$

$$= x \sin x + \cos x + C$$

$$\int \ln x \, dx = \int \underset{g'(x)}{(\ln x)} \underset{f(x)}{(1)} \, dx$$

$$\int f(x) g'(x) = f(x) g(x) - \int f'(x) g(x) \, dx$$

$f(x) = \ln x$

$f'(x) = \frac{1}{x} + c$

$g'(x) = 1$

$g(x) = x + c$

$$= x \ln(x) - \left[\int \frac{1}{x} \cdot x \, dx\right]$$

$$= x \ln x - x + c$$

$$= x(\ln x - 1) + c$$

$$\int x^2 e^x \, dx = x^2 e^x - \int 2x e^x \, dx$$

$f(x) = x^2$

$f'(x) = 2x$

$g(x) = e^x$

$g'(x) = e^x$

$$\int 2x e^x \, dx = 2x e^x - \int 2 e^x \, dx$$

$f(x) = 2x$

$f'(1) = 2$

$g(x) = e^x$

$g'(x) = e^x$

$$= 2x e^x - 2e^x + c$$

$$x^2 e^x - \left[2x e^x - 2e^x + c\right]$$

$$x^2 e^x - 2x e^x + 2e^x + c$$

$$\int x^2 \ln x \, dx = f(x)\,g(x) - \int f'(x)\,g(x)$$

$f(x) = x^2$
$f'(x) = 2x$
$g(x) = \frac{1}{x}$
$g'(x) = \ln x$

$$= \frac{x^2}{x} - \int 2x \frac{1}{x} dx$$

$$= x - 2x + C$$

$f(x) = \ln x$
$f'(x) = \frac{1}{x}$
$g(x) = \frac{1}{3}x^3$
$g'(x) = x^2$

$$= \frac{\ln(x) x^3}{3} - \int \frac{1}{x}\left(\frac{x^3}{3}\right) dx$$

$$\frac{x^3 \ln(x)}{3} - \frac{1}{3}\int x^2 \, dx$$

$$\frac{x^3 \ln(x)}{3} - \frac{1}{9}x^3 + C$$

$f'(x) \ln x \to \frac{1}{x}$

$\int \ln x = x \ln(x) - x + C$

not same

$f(x) = \frac{1}{x}$

$f'(x) = \ln|x| + C$