Review Finding horizontal asymptotes . . .

$$f(x) = \frac{ax^n + \ldots}{bx^m + \ldots}$$ ← nth degree polynomial ← mth degree polynomial

$\boxed{1}$ If $n < m$, then the x-axis is the horizontal asymptote. $y = 0$

$\boxed{2}$ If $n = m$, then the horizontal asymptote is the line $y = \frac{a}{b}$

$\boxed{3}$ If $n > m$, then there is no horizontal asymptote. (There is a slant diagonal or oblique asymptote.)

Example 1 Find each limit.

1. $\lim_{x\to\infty}\left(\frac{x^2+1}{x^3+2x}\right) = 0$

2. $\lim_{x\to-\infty}\left(\frac{3x+1}{x^2}\right) = 0$

3. $\lim_{x\to\infty}\left(\frac{5x^2+2x-3}{9x^{23}+x+4}\right) = 0$

4. $\lim_{x\to\infty}\left(\frac{4x^2+1}{2x}\right) =$ does not exist

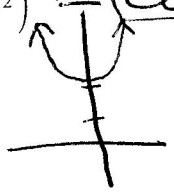
direction is ∞

5. $\lim_{x\to-\infty}\left(\frac{4x^2+1}{2x}\right) =$ does not exist

direction is $-\infty$

6. $\lim_{x\to\infty}\frac{(2x+1)(x-7)}{(3x-2)(4x+1)} = \lim_{x\to\infty}\frac{2x^2-13x-7}{12x^2-5x-2} = \frac{2}{12} = \frac{1}{6}$

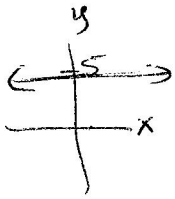
7. $\lim_{x\to\infty}(2+x^2) = \boxed{\infty}$

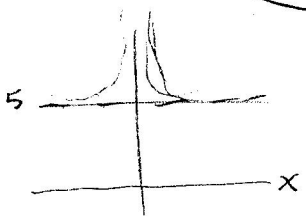


8. $\lim_{x\to\infty}\left(\frac{7}{x^2}+5\right) = \boxed{5}$

$\lim_{x\to\infty}\frac{7}{x^2} + \lim_{x\to\infty} 5$

$0 + 5$





Example 2 Draw a function if . . .

$f(0) = 3$ point $(0,3)$

$f(1) = 1$ point $(1,1)$

$\lim_{x\to\infty} f(x) = 0$

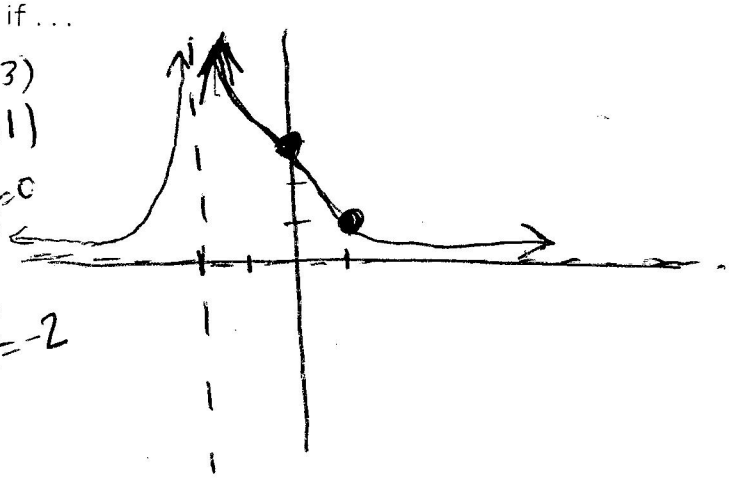
$\lim_{x\to-\infty} f(x) = 0$

h.a. at $y=0$

$\lim_{x\to-2^+} f(x) = \infty$

$\lim_{x\to-2^-} f(x) = \infty$

v.a. at $x=-2$



$f(0) = 0$ $(0,0)$

there is a removeable discontinuity at $x = 4$ → hole

$\lim_{x\to\infty} f(x) = 1$

$\lim_{x\to-\infty} f(x) = -1$

h.a. $y=1$ $y=-1$

$\lim_{x\to 4} f(x) = 2$

$\lim_{x\to-3} f(x) =$ *does not exist*