

Definition of a Polynomial Function

Let n be a nonnegative integer and let $a_n, a_{n-1}, \dots, a_2, a_1, a_0$ be real numbers, with $a_n \neq 0$. The function defined by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

is called a polynomial function of degree n . The number a_n , the coefficient of the variable to the highest power, is called the leading coefficient.

Example 1 Determine whether each of the following is a polynomial function.

a) $f(x) = 5x^{-1}$

NO
negative exponent

b) $g(x) = 4x^2 + ex - 10$

yes

c) $h(x) = -3x^\pi + 4x^3 + 11x$

NO

π is not an integer

d) $j(x) = 2x^{\frac{1}{2}} + 10x + 6$

NO
 $\frac{1}{2}$ is not an int.

e) $k(x) = \frac{1}{4}x^4 - 9x^3 + 7$

yes

f)

NO
imaginary Coefficient

Classifying Polynomials by Degree		
Name	Degree	Example
Constant	0	-9
Linear	1	$x - 4$
Quadratic	2	$x^2 + 3x - 1$
Cubic	3	$x^3 + 2x^2 + x + 1$
Quartic	4	$2x^4 + x^3 + 3x^2 + 4x - 1$
Quintic	5	$7x^5 + x^4 - x^3 + 3x^2 + 2x - 1$

****There are 2 polynomial functions you should be very familiar with... Linear & Quadratic****

LINEAR

Slope-Intercept Form: $y = mx + b$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Standard Form: $Ax + By = C$
A is positive, no fractions

Slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

QUADRATIC

Vertex Form: $y = a(x - h)^2 + k$ (h, k) vertex

Standard Form: $y = ax^2 + bx + c$

Finding the Vertex:

(h, k) or $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Completing the Square:

puts the eqn. in vertex form

Ways to solve quadratics:

graph \rightarrow x-intercept(s) quadratic formula
factor sq. roots complete the sq.

Example 2 Write a linear function that satisfies all of the following conditions:

- a) passes through the points $(0, 3)$ and $(-4, -1)$

$$y = mx + b \quad \text{y-intercept } m = \frac{-1 - 3}{-4 - 0} = \frac{-4}{-4} = 1$$

$$y = x + 3$$

- b) $f(4) = -2$ and $f(-4) = -4$

$$m = \frac{-4 - (-2)}{-4 - 4} = \frac{-2}{-8} = \frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = \frac{1}{4}(x - 4)$$

Example 3 Write an equation for the quadratic function with the given vertex and point:

- a) vertex $(2, 0)$ passing through $(1, 3)$

$$y = a(x - h)^2 + k$$

$$3 = a(1 - 2)^2 + 0$$

$$3 = a$$

- b) vertex $(-3, 4)$ passing through $(0, 0)$

$$0 = a(0 - (-3))^2 + 4$$

$$0 = 9a + 4$$

$$-4 = 9a \quad a = -\frac{4}{9}$$

$$y = 3(x - 2)^2 + 0$$

$$y = -\frac{4}{9}(x + 3)^2 + 4$$

To convert a quadratic from standard form to vertex form:

Step 1: Check the coefficient of the x^2 term. If 1, go to step 2.

If not 1, factor out the coefficient from x^2 and x terms.

Step 2: Calculate the value of $(b/2)^2$

Step 3: Group the x^2 and x term together, then add $(b/2)^2$ and subtract $(b/2)^2$

Step 4: Factor & Simplify

Example 4 Use completing the square to write the following equations in vertex form:

a) $y = x^2 + 6x - 11$

$$\left(\frac{6}{2}\right)^2 = 9$$

$$y = x^2 + 6x + 9 - 11 - 9$$

$$y = (x + 3)^2 - 20$$

b) $y = 2x^2 - 12x + 1$

$$y = 2(x^2 - 6x + 9) + 1 - 18$$

$$\left(\frac{-6}{2}\right)^2 = 9$$

$$y = 2(x - 3)^2 - 17$$

c) $y = -x^2 - 3x - 5$

$$y = -1(x^2 + 3x + 9) - 5 - 9$$

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$y = -(x + \frac{3}{2})^2 - \frac{11}{4}$$

d) $y = \frac{1}{3}x^2 - 4x - 1$

$$y = \frac{1}{3}(x^2 - 12x + 36) - 1 - 36$$

$$\left(\frac{-12}{2}\right)^2 = 36$$

$$y = \frac{1}{3}(x - 6)^2 - 13$$