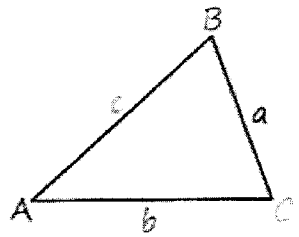
The Law of Cosines—used when given 3 sides OR 2 sides and an included angle (SSS or SAS)



Law of Cosines

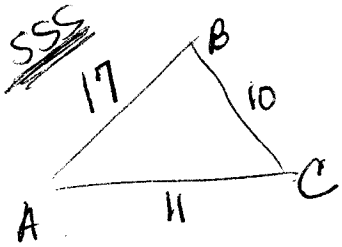
$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Example 1 Solve ΔABC: a = 10, b = 11, and c = 17

SSS



$c^2 = a^2 + b^2 - 2ab\cos C$

$(17)^2 = (10)^2 + (11)^2 - 2(10)(11)\cos C$

$289 = 100 + 121 - 220\cos C$

$68 = -220\cos C$

$C = \cos^{-1}\left(\frac{68}{-220}\right) = 108.0^\circ$

$b^2 = a^2 + c^2 - 2ac\cos B$

$(11)^2 = (10)^2 + (17)^2 - 2(10)(17)\cos B$

$121 = 100 + 289 - 340\cos B$

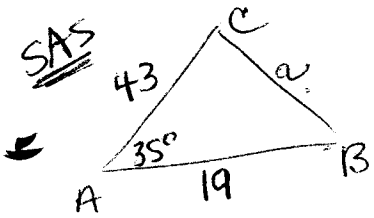
$-268 = -340\cos B$

$B = \cos^{-1}\left(\frac{268}{340}\right) = 38.0^\circ$

$A = 34^\circ$

Example 2 Solve ΔABC: A = 35°, b = 43, and c = 19

SAS



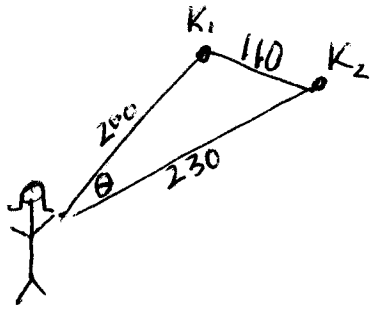
$a^2 = (43)^2 + (19)^2 - 2(43)(19)\cos 35^\circ$

$a = 29.5$

$(43)^2 = (29.5)^2 + (19)^2 - 2(29.5)(19)\cos B$

$B = 123.4^\circ$ $C = 21.6^\circ$

Example 3 A girl is flying two kites at the same time. If the strings are 200 ft and 230 ft long, and the kites are 110 ft apart, what angle do the strings in her hand make?



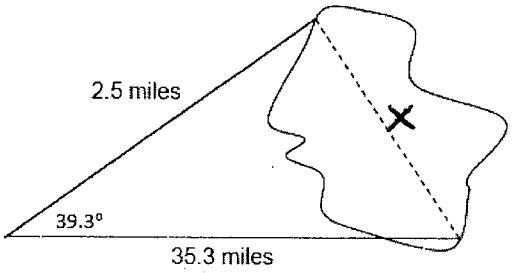
$110^2 = 200^2 + 230^2 - 2(200)(230)\cos\theta$

$-80800 = -92000\cos\theta$

$\theta = \cos^{-1}\left(\frac{80800}{92000}\right)$

$\boxed{\theta = 28.6^\circ}$

Example 4 To find the distance across a small lake, a surveyor has taken the measurements shown. Find the distance across the lake using this information.

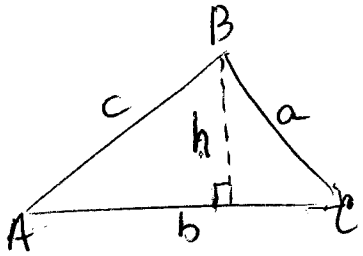


$x^2 = 2.5^2 + 35.3^2 - 2(2.5)(35.3)\cos 39.3^\circ$

$\boxed{x = 33.4 \text{ mi}}$

Area of a triangle

$\Delta\text{Area} = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$



$\sin C = \frac{h}{a}$

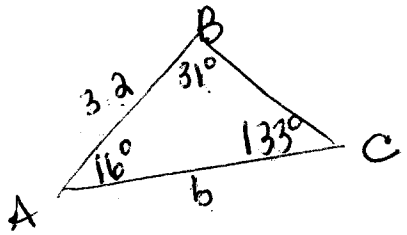
$h = a\sin C$

$\frac{1}{2} \cdot b \cdot a\sin C$

Example 5 Find the area of a triangle with sides of length 5 and 12 and included angle 40°.

$A = \frac{1}{2}(5)(12)\sin 40^\circ = \boxed{19.3 \text{ units}^2}$

Example 6 Find the area of triangle ABC to the nearest thousandth if c = 3.2, A = 16°, and B = 31°.



$\frac{\sin 133^\circ}{3.2} = \frac{\sin 31^\circ}{b}$

$b\sin 133^\circ = 3.2\sin 31^\circ$

$b = \frac{3.2\sin 31^\circ}{\sin 133^\circ} = 2.254$

$A = \frac{1}{2}(3.2)(2.254)\sin 16^\circ$

$\boxed{A = .994 \text{ units}^2}$

Heron's Formula

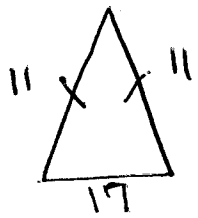
Let a, b and c be the sides of ΔABC, let s denote the **semiperimeter** $(a+b+c)/2$. Then the area of ΔABC is given by Area = $\sqrt{s(s-a)(s-b)(s-c)}$

Example 7 Find the area of triangle ABC to the nearest hundredth if a = 2, b = 7, and c = 8.

$s = \frac{2+7+8}{2} = 8.5$

$A = \sqrt{8.5(8.5-2)(8.5-7)(8.5-8)} = \boxed{6.44 \text{ units}^2}$

Example 8 Find the area of an isosceles triangle with a perimeter of 39 and a base of length 17 inches.



$s = \frac{11+11+17}{2} = 19.5$

$A = \sqrt{19.5(19.5-11)(19.5-11)(19.5-17)} = \boxed{59.3 \text{ in}^2}$