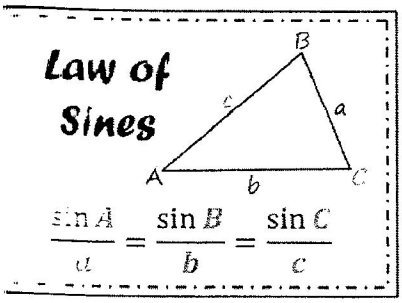
[redacted] Unit 6 Notes—Law of Sines

[redacted] Sines—used when given 2 angles and a side OR 2 consecutive sides and a non-included angle. ASA, AAS, or SSA.

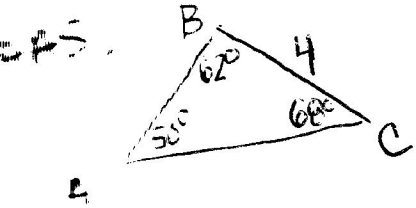


Law of Sines

AAS	SSA	ASA
ONE SOLUTION	AMBIGUOUS CASE	ONE SOLUTION
Domain error NO SOLUTION	Second angle option violates triangle angle-sum theorem ONE SOLUTION	Both angles satisfy the angle angle-sum theorem TWO SOLUTIONS

Example 1 Solve ΔABC: A = 50°, B = 62°, and a = 4.

AAS



$\frac{\sin 50^\circ}{4} = \frac{\sin 62^\circ}{b}$

$b \cdot \sin 50^\circ = 4 \sin 62^\circ$

$b = \frac{4 \sin 62^\circ}{\sin 50^\circ} \approx 4.61$

$\frac{\sin 50^\circ}{4} = \frac{\sin 68^\circ}{c}$

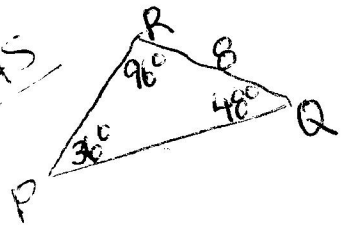
$c \cdot \sin 50^\circ = 4 \sin 68^\circ$

$c = \frac{4 \sin 68^\circ}{\sin 50^\circ} = 4.84$

$\boxed{\begin{array}{l} C = 68^\circ \\ b = 4.61 \\ c = 4.84 \end{array}}$

Example 2 Solve ΔPQR: P = 36°, Q = 48°, and p = 8.

AAS



$\frac{\sin 36^\circ}{8} = \frac{\sin 96^\circ}{r}$

$r \cdot \sin 36^\circ = 8 \sin 96^\circ$

$r = \frac{8 \sin 96^\circ}{\sin 36^\circ} = 13.54$

$\frac{\sin 36^\circ}{8} = \frac{\sin 48^\circ}{q}$

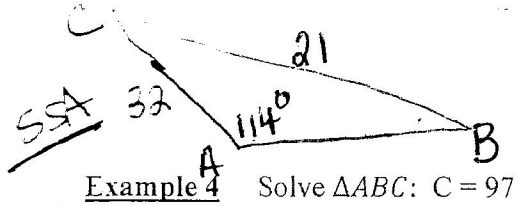
$q \cdot \sin 36^\circ = 8 \sin 48^\circ$

$q = \frac{8 \sin 48^\circ}{\sin 36^\circ} = 10.11$

$\boxed{\begin{array}{l} R = 96^\circ \\ r = 13.54 \\ q = 10.11 \end{array}}$

Example 3 Solve ΔABC: A = 114°, a = 21, and b = 32.

SSA



$\frac{\sin 114^\circ}{21} = \frac{\sin B}{32}$

$21 \sin B = 32 \sin 114^\circ$

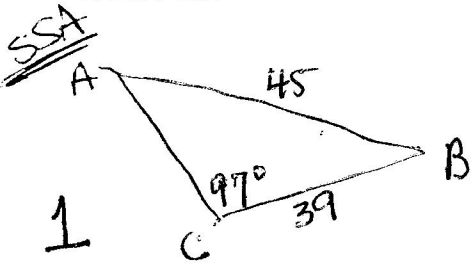
$\sin B = \frac{32 \sin 114^\circ}{21}$

$B = \sin^{-1}\left(\frac{32 \sin 114^\circ}{21}\right)$

$\boxed{\text{no soln}}$

Example 4 Solve ΔABC: C = 97°, c = 45, and a = 39.

SSA



1

$\frac{\sin 97^\circ}{45} = \frac{\sin A}{39}$

$45 \sin A = 39 \sin 97^\circ$

$A = \sin^{-1}\left(\frac{39 \sin 97^\circ}{45}\right)$

$A = 59.3^\circ$

$B = 180^\circ - 97^\circ - 59.3^\circ = 23.7^\circ$

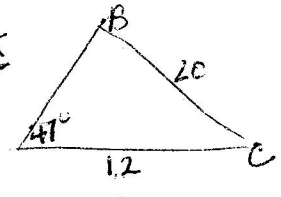
$\frac{\sin 23.7^\circ}{b} = \frac{\sin 97^\circ}{45}$

$b = \frac{45 \sin 23.7^\circ}{\sin 97^\circ} = 18.19$

$\boxed{\begin{array}{l} A = 59.3^\circ \\ B = 23.7^\circ \\ b = 18.19 \end{array}}$

Example 5 Solve ΔABC: A = 47°, a = 20, and b = 12.

SSA



$\frac{\sin 47^\circ}{20} = \frac{\sin B}{12}$

$B = \sin^{-1}\left(\frac{12 \sin 47^\circ}{20}\right)$

$B = 26.0^\circ$

$C = 180^\circ - 26.0^\circ - 47^\circ = 106.97^\circ$

$\frac{\sin 47^\circ}{20} = \frac{\sin 106.97^\circ}{c}$

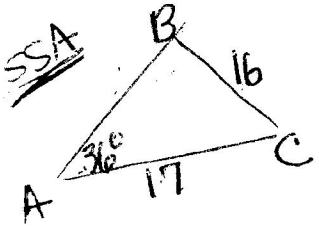
$c = \frac{20 \sin 106.97^\circ}{\sin 47^\circ}$

$c = 26.16$

$B = 26.0^\circ$
$C = 107.0^\circ$
$c = 26.16$

Example 6 Solve ΔABC: A = 36°, b = 17, and a = 16.

SSA



$\frac{\sin 36^\circ}{16} = \frac{\sin B}{17}$

$B = \sin^{-1}\left(\frac{17 \sin 36^\circ}{16}\right)$

$B = 38.6^\circ$ OR $B = 180^\circ - 38.6^\circ = 141.4^\circ$

$C = 180^\circ - 36^\circ - 38.6^\circ = 105.4^\circ$

$\frac{\sin 36^\circ}{16} = \frac{\sin 105.4^\circ}{c}$

$c = \frac{16 \sin 105.4^\circ}{\sin 36^\circ} = 26.25$

$C = 180^\circ - 36^\circ - 141.4^\circ = 2.6^\circ$

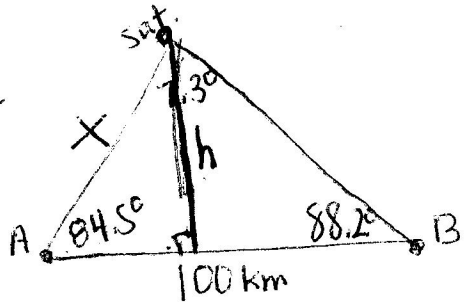
$\frac{\sin 36^\circ}{16} = \frac{\sin 2.6^\circ}{c}$

$c = \frac{16 \sin 2.6^\circ}{\sin 36^\circ} = 1.26$

$B = 38.6^\circ$	$B = 141.4^\circ$
$C = 105.4^\circ$	$C = 2.6^\circ$
$c = 26.25$	$c = 1.26$

Example 7 A satellite passes over two tracking stations, *A* and *B*, 100 km apart. When the satellite is between the two stations the angle of elevation at the stations are measured as 84.5° and 88.2° respectively. What is the distance between the satellite and station *A*.? How high is the satellite off the ground?

ASA



$\frac{\sin 7.3^\circ}{100} = \frac{\sin 88.2^\circ}{x}$

$x = \frac{100 \sin 88.2^\circ}{\sin 7.3^\circ}$

$x = 786.61$ km

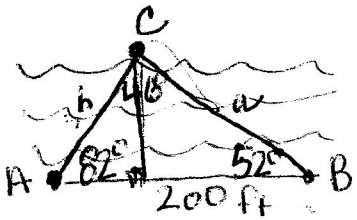
$\sin 84.5^\circ = \frac{h}{786.61}$

$h = 786.61 \cdot \sin 84.5^\circ$

$h = 782.99$ km

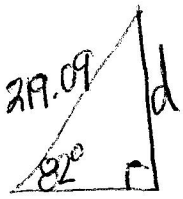
Example 8 To find the distance across a river, a surveyor chooses points *A* and *B*, which are 200 ft. apart on one side of the river. She chooses a reference point *C* on the opposite side of the river and finds that $< BAC = 82°$ and $< ABC = 52°$. Find the distance across the river.

ASA



$\frac{\sin 46^\circ}{200} = \frac{\sin 52^\circ}{b}$

$b = \frac{200 \sin 52^\circ}{\sin 46^\circ} = 219.09$



$\sin 82^\circ = \frac{d}{219.09}$

$d = 219.09 \sin 82^\circ = 216.96$ ft