

Local & Absolute Extrema

local/relative maximums and minimums

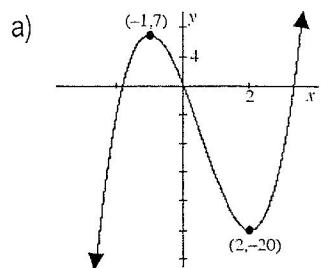
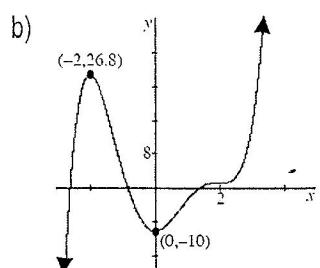
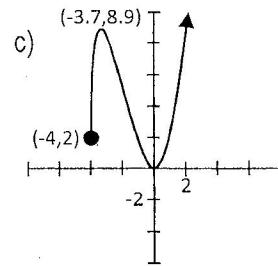
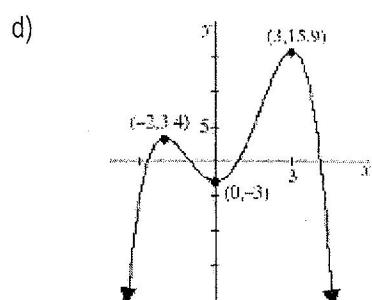
peaks

valley

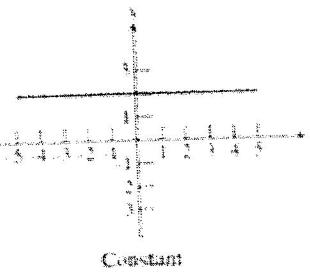
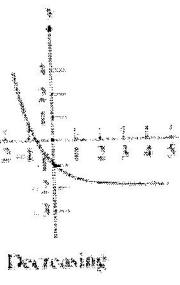
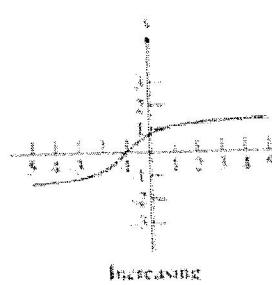
occur at turns in the graph
and sometimes endpointsglobal/absolute maximums and minimumsabs. max = the highest y-value
abs. min = the lowest y-value

things to remember

- not all functions have maximums and/or minimums
- a function may have a relative max/min but not an absolute max/min
- y-values should be given unless the problem asks for the x-value at which the max/min occurs

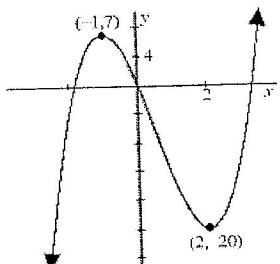
Example 1 Identify the local and absolute maximums and minimums for each of the functions:Local minimum: -20Local maximum: 7Absolute minimum: noneAbsolute maximum: noneLocal minimum: -10Local maximum: 26.8Absolute minimum: noneAbsolute maximum: noneLocal minimum: 2Local maximum: 7.8Absolute minimum: 0Absolute maximum: noneLocal minimum: -3Local maximum: 3.4, 15.9Absolute minimum: noneAbsolute maximum: 15.9

Increasing & Decreasing Intervals



Example 2 Identify the interval(s) over which the following functions are increasing and decreasing.

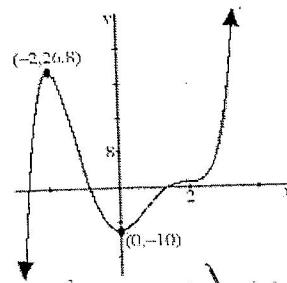
a)



increasing: $(-\infty, -1) \cup (2, \infty)$

decreasing: $(-1, 2)$

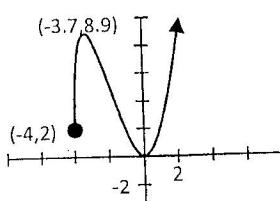
b)



increasing: $(-\infty, -2) \cup (0, \infty)$

decreasing: $(-2, 0)$

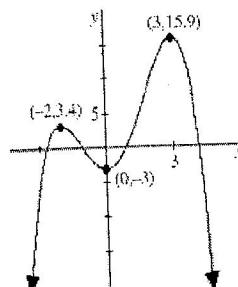
c)



increasing: $(-4, -3.7) \cup (0, \infty)$

decreasing: $(-3.7, 0)$

d)



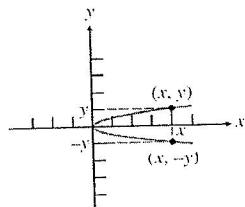
increasing: $(-\infty, -2) \cup (0, 3)$

decreasing: $(-2, 0) \cup (3, \infty)$

Symmetry

Symmetry with respect to the x-axis "NEITHER"

Graphically

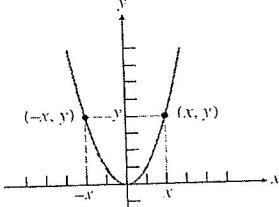


Algebraically

Graphs with this kind of symmetry are not functions (except the zero function), but we can say that $(x, -y)$ is on the graph whenever (x, y) is on the graph.

Symmetry with respect to the y-axis "EVEN"

Graphically



Algebraically

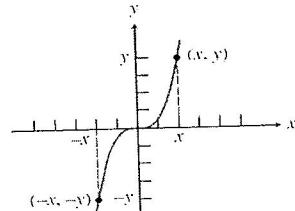
For all x in the domain of f ,

$$f(-x) = f(x)$$

Functions with this property (for example, x^n , n even) are **even** functions.

Symmetry with respect to the origin "ODD"

Graphically



Algebraically

For all x in the domain of f ,

$$f(-x) = -f(x)$$

Functions with this property (for example, x^n , n odd) are **odd** functions.

Example 3 Determine algebraically if each of the following functions is even, odd, or neither.

a) $f(x) = x^2 - 3$

$$f(-x) = (-x)^2 - 3 \\ = x^2 - 3 \\ \text{Same. EVEN}$$

d) $f(x) = -2x^4 \sqrt{x+3}$

$$f(-x) = -2(-x)^4 \sqrt{-x+3} \\ = -2x^4 \sqrt{-x+3} \\ \text{NEITHER}$$

b) $g(x) = x^2 - 2x - 2$

$$g(-x) = (-x)^2 - 2(-x) - 2 \\ = x^2 + 2x - 2 \\ \text{NEITHER}$$

e) $g(x) = 7x^5 - 4x^3 + 11x$

$$g(-x) = 7(-x)^5 - 4(-x)^3 + 11(-x) \\ = -7x^5 + 4x^3 - 11x \\ \text{opposite ODD}$$

c) $h(x) = \frac{x^3}{4-x^2}$

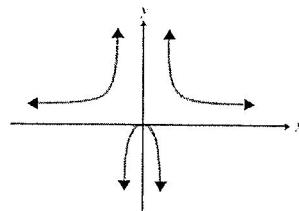
$$h(-x) = \frac{(-x)^3}{4-(-x)^2} = \frac{-x^3}{4-x^2} \\ \text{opposite ODD}$$

f) $h(x) = \frac{x^3}{4x-x^5}$

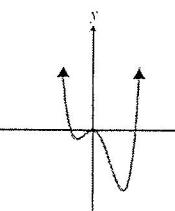
$$h(-x) = \frac{(-x)^3}{4(-x)-(-x)^5} = \frac{-x^3}{-4x+x^5} \\ = \frac{-x^3}{-(4x-x^5)} = \frac{x^3}{4x-x^5} \\ \text{same EVEN}$$

Boundedness

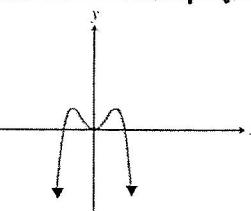
- ❖ A function is "BOUNDED BELOW" if it has an absolute minimum
- ❖ A function is "BOUNDED ABOVE" if it has an abs. max
- ❖ A function is "BOUNDED" if it has an abs. max & min
- ❖ A function is "UNBOUNDED" if it is not bounded above or below



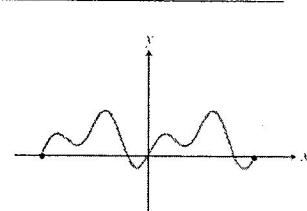
Not bounded above
Not bounded below



Not bounded above
Bounded below



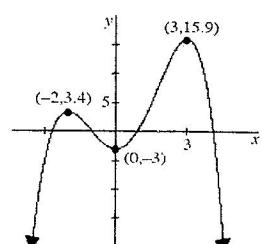
Bounded above
Not bounded below



Bounded

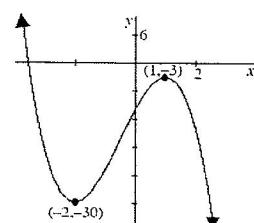
Example 4 Determine the boundedness of the functions below:

a)



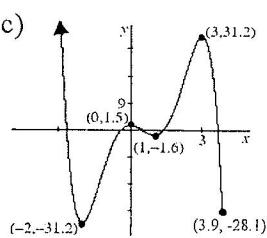
bounded above

b)



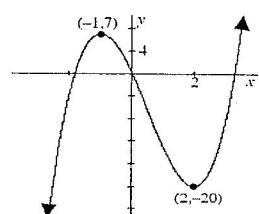
unbounded

c)



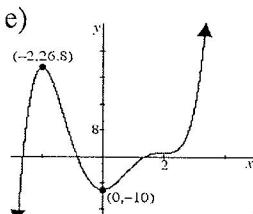
bounded below

d)



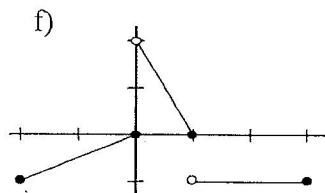
unbounded

e)



unbounded

f)



bounded