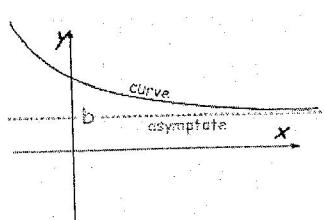
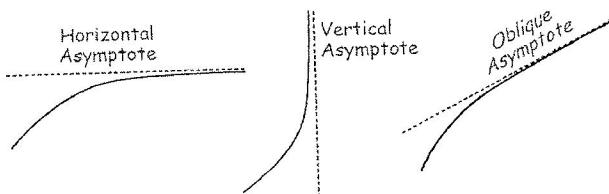


## Asymptotes

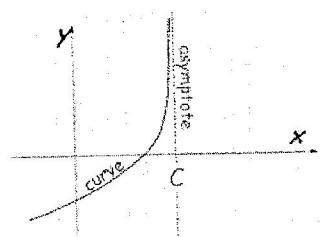
asymptote — a line that the graph of a function approaches  
 - the graph usually doesn't cross it

## Types of Asymptotes



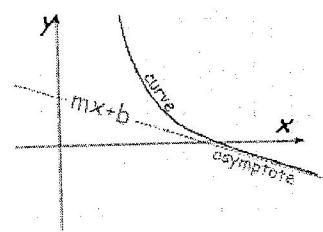
It is a Horizontal Asymptote when:

as  $x$  goes to infinity (or  $-\infty$ ) the curve approaches some constant value  $b$



It is a Vertical Asymptote when:

as  $x$  approaches some constant value  $c$  (from the left or right) then the curve goes towards infinity (or  $-\infty$ ).



It is an Oblique Asymptote when:

as  $x$  goes to infinity (or  $-\infty$ ) then the curve goes towards a line  $y = mx + b$

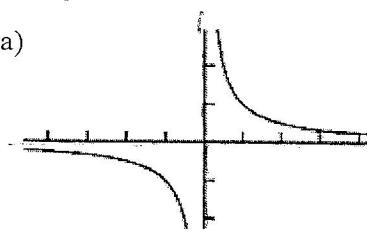
(note:  $m$  is not zero as that is a Horizontal Asymptote).

## How to find the asymptotes

vertical	<b>set the denominator equal to zero and solve (if possible)</b> <ul style="list-style-type: none"> <li>the zeroes (if any) are the vertical asymptotes (assuming no cancellation)</li> <li>everything else is in the domain</li> </ul>
horizontal	<b>compare the degrees of the numerator and the denominator</b> <ul style="list-style-type: none"> <li>When the degree of the denominator is <math>&gt;</math> than the degree of the numerator, the horizontal asymptote will be at <math>y = 0</math>.</li> <li>When the degree of the numerator is <math>=</math> to the degree of the denominator, the horizontal asymptote will be at <math>y = \frac{\text{leading coefficient of the numerator}}{\text{leading coefficient of the denominator}}</math></li> <li>When the degree of the numerator is <math>&gt;</math> than the degree of the denominator, there is no horizontal asymptote but there is an oblique asymptote.</li> </ul>

Example 1 Determine the horizontal and vertical asymptotes of the following functions:

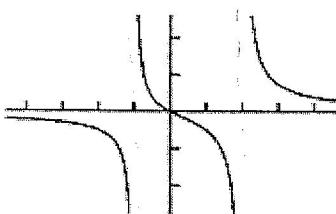
a)



HA:  $y = 1$

VA:  $x = 0$

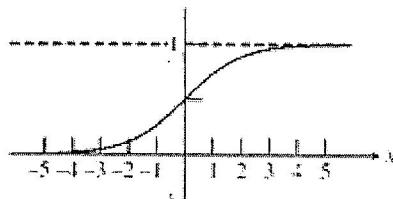
b)



HA:  $y = 1$

VA:  $x = -1, x = 1$

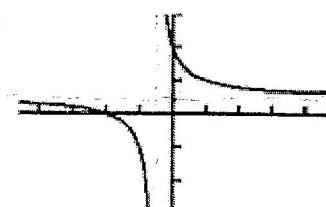
c)



HA:  $y = 1, y = -1$

VA: none

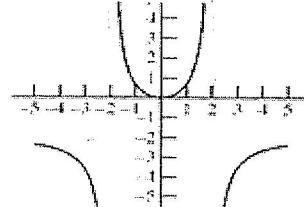
d)



HA:  $y = \frac{1}{2}$

VA:  $x = -\frac{1}{2}$

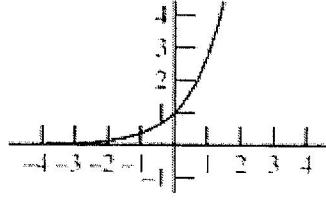
e)



HA:  $y = -2$

VA:  $x = -2, x = 2$

f)



HA:  $y = 4$

VA: none

g)  $y = \frac{x^2+4}{x+3}$

HA: none

VA:  $x = -3$

h)  $y = \frac{2x-3}{x^2+3x-4} = \frac{2x-3}{(x+4)(x-1)}$

HA:  $y = 0$

VA:  $x = -4, x = 1$

i)  $y = \frac{4x^2+4x}{2x^2-2} = \frac{4x(x+1)}{2(x^2-1)}$

HA:  $y = \frac{4}{2} = 2$

VA:  $x = 1$

$\frac{4x(x+1)}{2(x+1)(x-1)}$

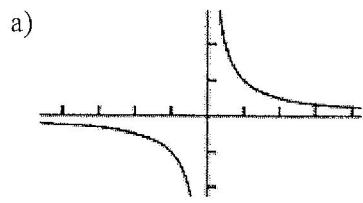
$x = -1$   
hole

End Behavior

Left End Behavior (LEB)--The left end behavior of a graph of the function  $f(x)$  describes the behavior of  $f(x)$  as  $x$  approaches  $-\infty$ .

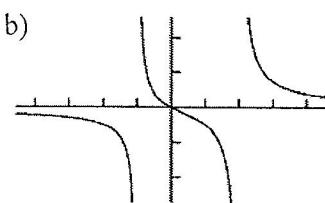
Right End Behavior (REB)--The right end behavior of a graph of the function  $f(x)$  describes the behavior of  $f(x)$  as  $x$  approaches  $\infty$ .

Example 2 State the left and right end behavior of the graphs below using limit notation:



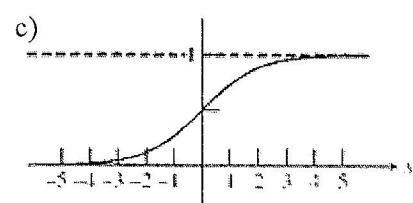
$$\text{LEB: } \lim_{x \rightarrow -\infty} f(x) = 0$$

$$\text{REB: } \lim_{x \rightarrow \infty} f(x) = 0$$



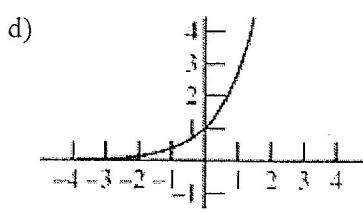
$$\text{LEB: } \lim_{x \rightarrow -\infty} f(x) = 0$$

$$\text{REB: } \lim_{x \rightarrow \infty} f(x) = 0$$



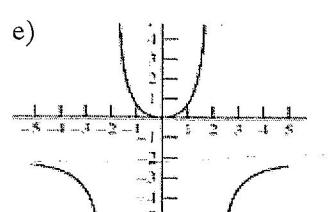
$$\text{LEB: } \lim_{x \rightarrow -\infty} f(x) = 0$$

$$\text{REB: } \lim_{x \rightarrow \infty} f(x) = 1$$



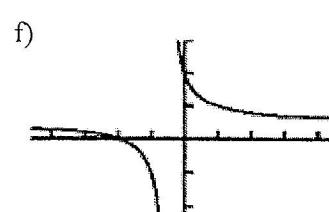
$$\text{LEB: } \lim_{x \rightarrow -\infty} f(x) = 0$$

$$\text{REB: } \lim_{x \rightarrow \infty} f(x) = \infty$$



$$\text{LEB: } \lim_{x \rightarrow -\infty} f(x) = -2$$

$$\text{REB: } \lim_{x \rightarrow \infty} f(x) = -2$$



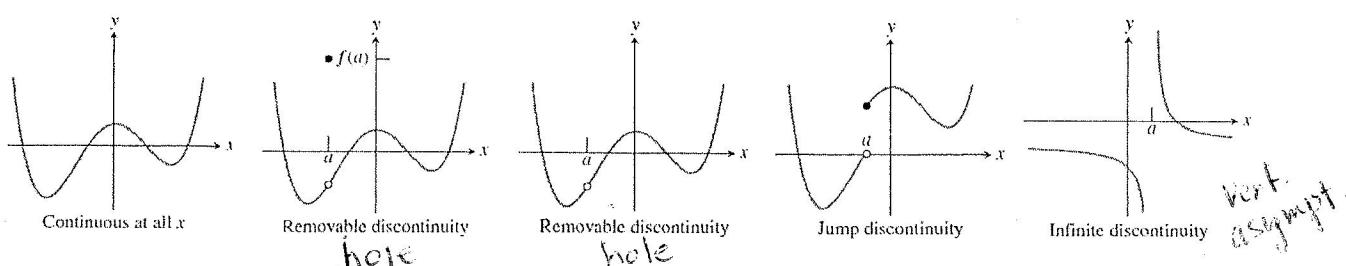
$$\text{LEB: } \lim_{x \rightarrow -\infty} f(x) = \frac{1}{2}$$

$$\text{REB: } \lim_{x \rightarrow \infty} f(x) = \frac{1}{2}$$

### Discontinuity

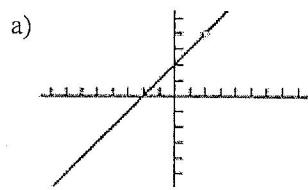
**Continuous Function**--a function where the graph does not come apart at any point on its domain.

- If a function is **not continuous**, then it could have one of the following: Removable Discontinuity, Jump Discontinuity, or Infinite Discontinuity.

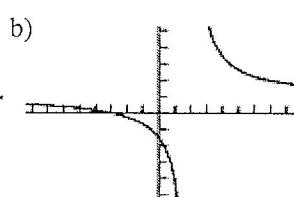


NOTE: Jump and infinite discontinuities are "Non-Removable".

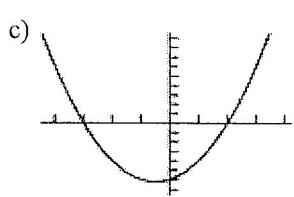
Example 3 Determine whether each function is continuous or discontinuous. If the function is discontinuous, state the type of discontinuity and whether the discontinuity is removable or non-removable.



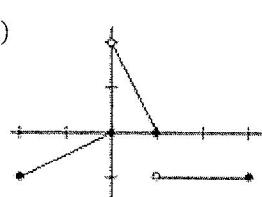
discont.  
hole  
rem.



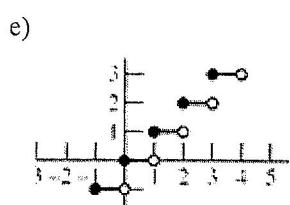
discont.  
infinite  
non-rem.



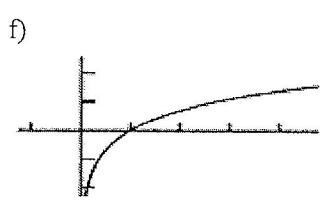
cont.



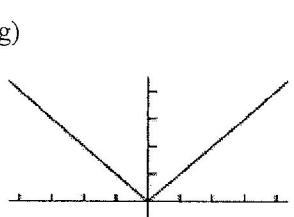
discont.  
jump  
non-rem.



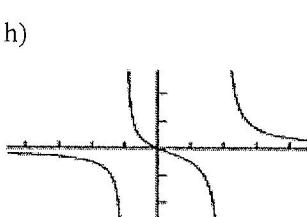
discont.  
jump  
non-rem.



cont.



cont.



discont.  
infinite  
non-rem.