



When dealing with simple **COMBINATIONS** of functions (such as  $f + g$ ,  $f - g$ ,  $f \cdot g$ , and  $f \div g$ ), the domain of the resulting function consists of all numbers that belong to BOTH the domain of  $f$  and the domain of  $g$ .

Example 1 Given that  $f(x) = \sqrt{2-x}$  &  $g(x) = \sqrt{2x+5}$ , determine the value of  $f+g$ ,  $f-g$ ,  $f \cdot g$ , and  $f \div g$  and their domains.

$$(f+g)(x) = \sqrt{2-x} + \sqrt{2x+5}$$

$$D: \left[-\frac{5}{2}, 2\right]$$

$$(f-g)(x) = \sqrt{2-x} - \sqrt{2x+5}$$

$$D: \left[-\frac{5}{2}, 2\right]$$

$$(f \cdot g)(x) = \sqrt{2-x} \cdot \sqrt{2x+5}$$

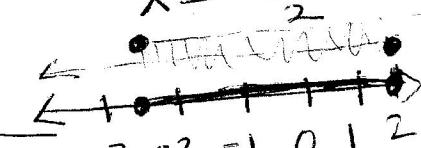
$$= \sqrt{(2-x)(2x+5)} = \sqrt{4x+10-2x^2-5x}$$

$$D: \left[-\frac{5}{2}, 2\right]$$

$$\frac{(f \div g)(x)}{\sqrt{2-x} \cdot \sqrt{2x+5}} = \frac{\sqrt{-2x^2-x+10}}{2x+5}$$

$$D: \left(-\frac{5}{2}, 2\right]$$

$$g: 2x+5 \geq 0 \\ 2x \geq -5 \\ x \geq -\frac{5}{2}$$



Example 2

Perform the indicated operation, simplify your result as much as possible, and then determine the domain of the resulting function.

a) If  $f(x) = \frac{4}{x-2}$  and  $g(x) = \frac{-x^2}{x-2}$ , find  $(f+g)(x)$ .

$$\frac{4}{x-2} + \frac{-x^2}{x-2} = \frac{4-x^2}{x-2}$$

$$D: (-\infty, 2) \cup (2, \infty)$$

b) If  $f(x) = \sqrt{x-10}$  and  $g(x) = \sqrt{x+10}$ , find  $(fg)(x)$ .

$$\sqrt{x-10} \cdot \sqrt{x+10} = \sqrt{(x-10)(x+10)} = \sqrt{x^2-100}$$

$$D: [10, \infty)$$

$$f: x-10 \geq 0 \\ x \geq 10$$

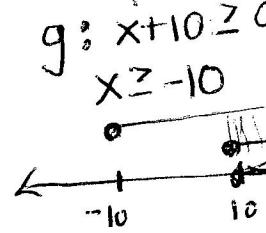
c) If  $f(x) = \sqrt{x-5}$  and  $g(x) = \sqrt{x+2}$ , find  $(f/g)(x)$ .

$$\frac{\sqrt{x-5}}{\sqrt{x+2}} \cdot \frac{\sqrt{x+2}}{\sqrt{x+2}} = \frac{\sqrt{x^2-3x-10}}{x+2}$$

$$D: [5, \infty)$$

$$f: x-5 \geq 0 \\ x \geq 5$$

$$g: x+2 \geq 0 \\ x \geq -2$$





When dealing with **COMPOSITIONS** of functions (such as  $f \circ g$ ), the domain of the function consists of all x-values in the domain of  $g$  that map to  $g(x)$  values in the domain of  $f$ .

Example 3 Given  $f(x) = \sqrt{x-1}$  and  $g(x) = x^2 + 1$ . Find each composition function and its domain.

$$\text{a) } f(g(x)) = \sqrt{x^2 + 1 - 1} = \sqrt{x^2} = |x|$$

$$f: (-\infty, \infty) \quad D: (-\infty, \infty)$$

$$\text{b) } g(f(x)) = (\sqrt{x-1})^2 + 1 = x-1+1 = x$$

$$D: [1, \infty)$$

Example 4 Given  $f(x) = 9 - x^2$  and  $g(x) = \sqrt{x}$ . Find each composition function and its domain.

$$\text{a) } f(g(x)) = 9 - (\sqrt{x})^2 = 9 - x$$

$$f: (-\infty, \infty) \quad D: [0, \infty)$$

$$g: x \geq 0$$

$$\text{b) } g(f(x)) = \sqrt{9-x^2} \quad D: [-3, 3]$$

$$\begin{array}{l} 9-x^2 \geq 0 \\ (3-x)(3+x) \geq 0 \\ 3-x=0 \quad 3+x=0 \end{array} \quad \begin{array}{c} x=3, -3 \\ - + + - \\ -3 \quad 3 \end{array}$$

Example 5 Given  $f(x) = \frac{1}{x+3}$  and  $g(x) = x^2 - 3$ . Find each composition function and its domain.

$$\text{a) } f(g(x)) = \frac{1}{x^2-3+3} = \frac{1}{x^2}$$

$$f: (-\infty, -3) \cup (-3, \infty)$$

$$g: (-\infty, \infty)$$

$$\text{b) } g(f(x)) = \left(\frac{1}{x+3}\right)^2 - 3 = \frac{1}{(x+3)^2} - 3$$

$$D: (-\infty, -3) \cup (-3, \infty)$$

 When DECOMPOSING functions the purpose is to create two functions (not using the identity function) such that their composition is the given function. In other words, when given  $h(x)$  the goal is to define  $f(x)$  &  $g(x)$  so that  $h(x) = f(g(x))$ .

Example 6 For each  $h(x)$ , find the functions  $f$  and  $g$  such that  $h(x) = f(g(x))$ .

a)  $h(x) = (\underline{x+1})^2 - 3(\underline{x+1}) + 4$

$$g(x) = x+1$$

$$f(x) = x^2 - 3x + 4$$

b)  $h(x) = \sqrt[3]{2x+1}$

$$g(x) = 2x+1$$

$$f(x) = \sqrt[3]{x}$$

$$\left\{ \begin{array}{l} g(x) = 2x \\ f(x) = \sqrt[3]{x+1} \end{array} \right.$$

c)  $h(x) = \frac{x+5}{x^2+10x+25} = \frac{x+5}{(x+5)(x+5)}$

$$= \frac{x+5}{(x+5)^2} = \frac{1}{x+5}$$

$$g(x) = x+5$$

$$f(x) = \frac{x}{x^2} = \frac{1}{x}$$

d)  $h(x) = \underline{9x^2+6x-2}$

$$= 3(3x^2+2x) - 2$$

$$g(x) = 3x^2+2x$$

$$f(x) = 3x - 2$$