

RIGID TRANSFORMATION—a transformation that will leave the size and shape of a graph unchanged. This includes horizontal translations, vertical translations, reflections, or any combination of these.

NON-RIGID TRANSFORMATION—a transformation which will generally distort the shape of a graph. This includes horizontal or vertical stretches and shrinks.



Transformation	Function	Description
Horizontal Shift	$f(x + h)$	Shift left h units
	$f(x - h)$	Shift right h units
Vertical Shift	$f(x) + k$	Shift up k units
	$f(x) - k$	Shift down k units
Reflection	$-f(x)$	Reflect across x-axis
	$f(-x)$	Reflect across y-axis
Vertical Stretch/Compress	$a f(x), a > 1$	Stretch vertically by a factor of a
	$a f(x), 0 < a < 1$	Compress vertically by a factor of a
Horizontal Stretch/Compress	$f(ax), a > 1$	Compress horizontally by a factor of $\frac{1}{a}$
	$f(ax), 0 < a < 1$	Stretch horizontally by a factor of $\frac{1}{a}$

NOTE: If there is a coefficient to x and a horizontal translation (a “ b ” and an “ h ”) then the coefficient should be factored out in order to truly see what the horizontal shift is.



Order of function transformations

- Horizontal shifts
- Horizontal stretch/compression
- Reflection over y-axis
- Vertical stretch/compression
- Reflection over x-axis
- Vertical shifts

Example 1 Describe the transformation that occurs from a parent function.

- a) $f(x) = x^2 + 3$ $y = x^2$ shift up 3
- b) $f(x) = x^2 - 5$ $y = x^2$ shift down 5
- c) $f(x) = (x - 2)^2$ $y = x^2$ shift right 2
- d) $f(x) = (x + 4)^2$ $y = x^2$ shift left 4
- e) $f(x) = 3e^x$ $y = e^x$ vert. stretch by a factor of 3
- f) $f(x) = \frac{1}{2}e^x$ $y = e^x$ vert. shrink by a factor of $\frac{1}{2}$
- g) $f(x) = (3x)^2$ $y = x^2$ horiz. shrink by a factor of $\frac{1}{3}$
- h) $f(x) = \left(\frac{1}{2}x\right)^2$ $y = x^2$ horiz. stretch by a factor of 2
- i) $f(x) = -|x|$ $y = |x|$ reflect over x-axis
- j) $f(x) = |-x|$ $y = |x|$ reflect over y-axis

Example 2 Describe how the graph of $y = |x|$ can be transformed to the graph of the given equation.

- a) $y = |x| - 4$ shift down 4
- b) $y = |x + 2|$ shift left 2
- c) $y = -|x - 6|$ shift right 6, reflect over x-axis
- d) $y = |-x + 2| = |-1(x - 2)|$ shift right 2, reflect over y-axis
- e) $y = -|x + 3| - 7$ shift left 3
reflect over x-axis
down 7

Example 3 Find an equation for the following transformations of the function $f(x) = \sqrt{x}$.

- a) $f(x)$ is reflected over the y-axis and translated up 3 units

$$y = \sqrt{-x} + 3$$

- b) $f(x)$ is vertically stretched by a factor of 3 and translated 4 units left

$$y = 3\sqrt{x+4}$$

- c) $f(x)$ is horizontally shrunk by a factor of $\frac{1}{2}$ and reflected over the x-axis

$$y = -\sqrt{2x}$$

Example 4 Describe the following transformations that have been applied to one of the 12 parent functions.

a) $f(x) = 0.5 \sin(2x - 6) + 7 = 0.5 \sin[2(x - 3)] + 7$

horiz. shift right 3

horiz. shrink by a factor of $\frac{1}{2}$

vert. shrink by a factor of 0.5, shift up 7

b) $f(x) = -\ln(-x + 4) - 2 = -\ln[-1(x - 4)] - 2$

horiz. shift right 4

reflect over y-axis

reflect over x-axis, shift down 2

c) $f(x) = \frac{1}{1+e^x}$

reflect over y-axis

vert. stretch by a factor of 2

Example 5 Find an equation for the following transformations of the function $f(x) = e^x$.

- a) $f(x)$ is reflected over the x-axis and translated down 2 units

$$y = -e^x - 2$$

- b) $f(x)$ is vertically shrunk by a factor of $\frac{1}{4}$ and translated 6 units right

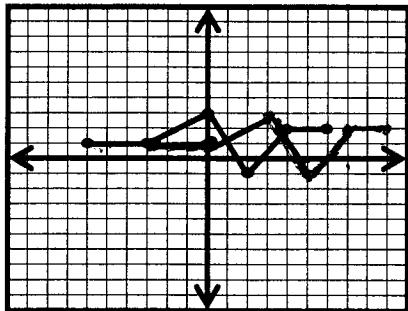
$$y = \frac{1}{4} e^{x-6}$$

- c) $f(x)$ is horizontally stretched by a factor of 7 and shifted up 3 and left 4

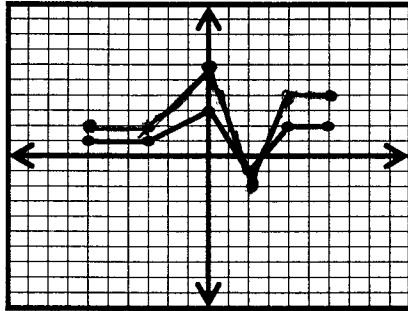
$$y = e^{\frac{1}{7}(x+4)} + 3$$

Example 6 Given the graph of $f(x)$ in each coordinate plane below, sketch each of the transformations indicated:

(a) $f(x-3)$ shift rt. 3

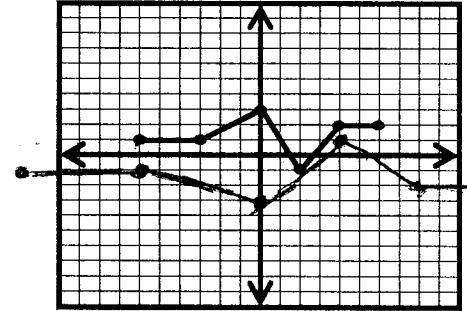


(b) $2f(x)$ vert. stretch * 2



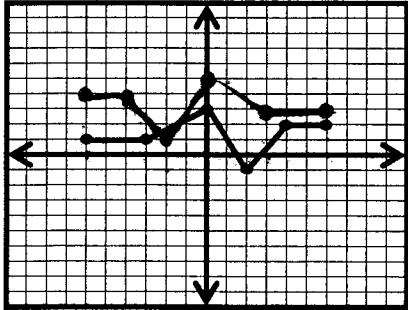
horiz. stretch * 2

(c) $-f(\frac{1}{2}x)$ reflect over x-axis



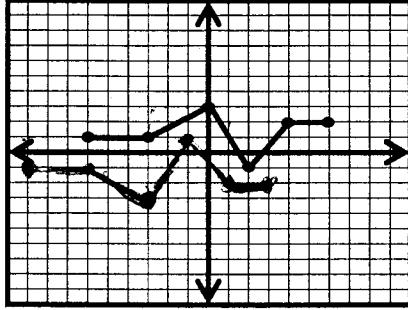
(d) $f(-x) + 2$ reflect over y-axis

Shift up 2



(e) $-f(x+3)$ reflect over x-axis

horiz. shift left 3



shift right 4

reflect over x-axis

(f) $-f(x-4) - 3$ Shift down 3

