6

AP Calculus BC

Notes 2.2 : n^{th} term test, geometric series test & telescoping series

Sequence: $a_1, a_2, a_3, a_4, \ldots, a_n$

Series: $a_1 + a_2 + a_3 + a_4 + \ldots + a_n = \sum_{n=1}^{\infty} a_n$

Partial Sums

$S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

.

.

$S_n = a_1 + a_2 + a_3 + \ldots + a_n$

Properties of Infinite Series

$\sum a_n = A \qquad \sum b_n = B$

1) $\sum c \cdot a_n = c \cdot A$

2) $\sum (a_n \pm b_n) = A \pm B$

Convergent/Divergent Series

In general, if the sequence of the partial sum, $\{S_n\}$, converges to "S", then the series,

$\sum_{n=1}^{\infty} a_n$, converges.

If $\{S_n\}$, diverges, then the series diverges.

Ex1) Determine the convergence of the series.

A) $\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \ldots$

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$

$\vdots$

$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} = \frac{2^n - 1}{2^n}$

$\lim_{n \to \infty} \frac{2^n - 1}{2^n} = 1$

B) $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + 4 + \ldots$

$S_1 = 1 = 1$

$S_2 = 1 + 2 = 3$

$S_3 = 1 + 2 + 3 = 6$

$\vdots$

$S_n = 1 + 2 + 3 + 4 + 5 + \ldots n = n + \sum_{i=1}^{n-1} i$

Diverges

C) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$

$S_1 = 1 - \frac{1}{2} = \frac{1}{2}$

$S_2 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} + \frac{1}{2}$

$S_3 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} + \frac{1}{6} + \frac{1}{2}$

$\vdots$

$S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots \frac{1}{n} - \frac{1}{n+1}$

limit is 1

thus converges.

telescoping

$(b_1 - b_2) + (b_2 - b_3) + (b_3 - b_4) + (b_4 - b_5) + \ldots$

$(1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + \ldots (\frac{1}{n} + \frac{-1}{n+1})$

$1 + \frac{-1}{n+1} \to 1 + \frac{-1}{\infty} \to 1$

GEOMETRIC SERIES TEST

Geometric Series have a common RATIO.

$\sum_{n=1}^{\infty} a(r)^{n-1}$ or $\sum_{n=0}^{\infty} a(r)^{n}$

$a + ar^1 + ar^2 + \cdots$

Sum of a Convergent Geometric Series

$$\boxed{\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}}$$

If $|r| < 1$, the series Converges.

If $|r| \geq 1$, the series Diverges.

Ex 2) Determine the convergence of the series.

A) $10 + 5 + \frac{5}{2} + \frac{5}{4} \cdots$

$a_1 = 10$ $a_n = 10(\frac{1}{2})^{n-1}$ $\sum_{n=1}^{\infty} ar^{n-1} = \frac{10}{1-\frac{1}{2}} = 20$

$r = \frac{1}{2}$

B) $-3 - 6 - 12 - 24 - \cdots$

$a_1 = -3$ $r = 2$ $a_n = -3(2)^{n-1}$ Diverges

C) $\sum_{n=0}^{\infty} \frac{1}{9}\left(\frac{1}{3}\right)^n$ $\frac{\frac{1}{9}}{1-\frac{1}{3}} = \frac{1}{9} \cdot \frac{3}{2} = \frac{1}{6}$

D) $\sum_{n=0}^{\infty} (-4)^n$ Diverges.

E) Look back to example 1A.

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$

$= 1$ Convergences

n^{th} Term Test (divergence)

Limit of the n^{th} term of a divergent series

If $\lim_{n \to \infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

Ex 3) Apply the n^{th} term test for divergence.

A) $\sum_{n=1}^{\infty} n^2 \to \lim_{n \to \infty} n^2 \Rightarrow \infty \to$ Diverges.

B) $\sum_{n=1}^{\infty} \frac{n+1}{n} \to \lim_{n \to \infty} \frac{n+1}{n} \Rightarrow \frac{\infty}{\infty} = 1$ Diverges

C) $\sum_{n=1}^{\infty} \frac{1}{n} \to \lim_{n \to \infty} \frac{1}{n} \to \frac{1}{\infty} = 0$ Convergent

D) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1} \to \lim_{n \to \infty} \frac{n!}{2n!+1} = \frac{1}{2}$ Divergent