

A polar coordinate system is a plane with a point O , the **pole**, and a ray from O , the **polar axis**. Each point P in the plane is assigned as **polar coordinates** as follows: r is the **directed distance** from O to P and θ is the **directed angle** whose initial side is on the polar axis and whose terminal side is on the line OP .

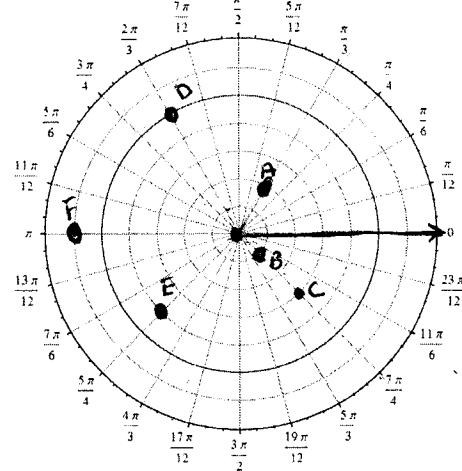
As in trigonometry, we measure θ as positive when moving counterclockwise and negative when moving clockwise. If $r > 0$, then P is on the terminal side of θ . If $r < 0$, then P is on the terminal side of $\theta + \pi$. We can use radian or degree measure for the angle θ .

points (r, θ)

EXAMPLE 1 Plotting Points in the Polar Coordinate System

Plot the points with the given polar coordinates.

- (a) $P(2, \pi/3)$
- (b) $Q(-1, 3\pi/4)$
- (c) $R(3, -45^\circ)$
- d) $(5, \frac{2\pi}{3})$
- e) $(-4, 45^\circ)$
- f) $(6, -\pi)$

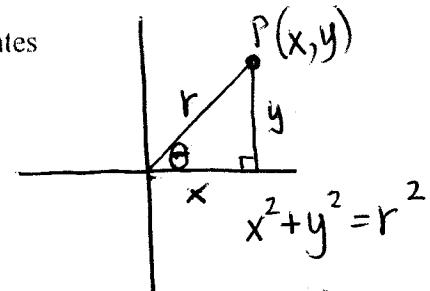


NOTE: Each polar coordinate pair determines a unique point. However, the polar coordinates of a point P in the plane are not unique.

Coordinate Conversion Equations

Let the point P have polar coordinates (r, θ) and rectangular coordinates (x, y) . Then

$$\begin{array}{l} P \rightarrow R \\ x = r \cos \theta, \\ y = r \sin \theta. \end{array} \quad \begin{array}{l} R \rightarrow P \\ r^2 = x^2 + y^2, \\ \tan \theta = \frac{y}{x}. \end{array}$$



EXAMPLE 2 Converting from Polar to Rectangular Coordinates

Find the rectangular coordinates of the points with the given polar coordinates.

- (a) $P(3, 5\pi/6)$
- (b) $Q(2, -200^\circ)$

$$\begin{aligned} x &= 3 \cdot \cos \frac{5\pi}{6} = 3 \cdot -\frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{2} \\ y &= 3 \cdot \sin \frac{5\pi}{6} = 3 \cdot \frac{1}{2} = \frac{3}{2} \end{aligned}$$

$$\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2} \right)$$

$$x = 2 \cos (-200^\circ)$$

$$y = 2 \sin (-200^\circ)$$

$$(-1.88, .68)$$

$$\cos \theta = \frac{x}{r}$$

$$\sin \theta = \frac{y}{r}$$

$$\tan \theta = \frac{y}{x}$$

EXAMPLE 3 Converting from Rectangular to Polar Coordinates

Find two polar coordinate pairs for the points with given rectangular coordinates.

- (a) $P(-1, 1)$
- (b) $Q(-3, 0)$

2nd quadrant

$$\tan \theta = \frac{1}{-1}$$

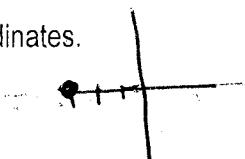
$$\begin{aligned} r^2 &= x^2 + y^2 \\ r^2 &= (-1)^2 + (1)^2 \end{aligned}$$

$$r = \sqrt{2}$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = \frac{3\pi}{4}$$

$$(\sqrt{2}, \frac{3\pi}{4}) \quad (\sqrt{2}, -225^\circ)$$



$$(3, \pi)$$

$$(-3, 0^\circ)$$

$$(3, -180^\circ)$$

Equation Conversion

We can use the Coordinate Conversion Equations to convert polar form to rectangular form and vice versa. Just as with parametric equations, the domain of a polar equation in r and θ is understood to be all values of θ for which the corresponding values of r are real numbers. You must also select a value for θ_{\min} and θ_{\max} to graph in polar mode.

Converting from Polar Form to Rectangular Form

EXAMPLE 4 Convert each polar equation to rectangular form.

A. $r = 4 \sec \theta$

$$r = \frac{4}{\cos \theta}$$

$$r \cos \theta = 4$$

$$\boxed{x = 4}$$

vertical line

B. $r = 4 \cos \theta$

$$r \cdot r = 4 \cdot r \cdot \cos \theta$$

$$x^2 + y^2 = 4x$$

circle

C. $r = 3 \cos \theta$

$$r^2 = 3r \cos \theta$$

$$\boxed{x^2 + y^2 = 3x}$$

D. $r^2 = -3 \sec \theta$

$$\frac{1}{r} \cdot r^2 = -\frac{3}{\cos \theta} \cdot \frac{1}{r}$$

$$r = \frac{-3}{r \cos \theta}$$

$$\sqrt{x^2 + y^2} = -\frac{3}{x}$$

$$x^2 + y^2 = \frac{9}{x^2}$$

E. $\left(\frac{r}{3 \tan \theta}\right) = (\sin \theta) \cdot r$

$$\frac{r^2}{3 \tan \theta} = r \sin \theta$$

$$\frac{x^2 + y^2}{3 \cdot \frac{y}{x}} = y$$

$$x^2 + y^2 = \frac{3y}{x} \cdot y$$

$$x^2 + y^2 = \frac{3y^2}{x}$$

$$x^3 + xy^2 = 3y^2$$

Converting from Rectangular Form to Polar Form

EXAMPLE 5 Convert each rectangular equation to polar form.

A. $x^2 + y^2 = 1$

$$r^2 = 1$$

$$r = \pm 1$$

$$r \sin \theta = 2r \cos \theta + 1$$

$$r \sin \theta - 2r \cos \theta = 1$$

$$r(\sin \theta - 2\cos \theta) = 1$$

$$r = \frac{1}{\sin \theta - 2\cos \theta}$$

C. $y = \frac{3}{x}$

$$r \sin \theta = \frac{3}{r \cos \theta}$$

$$r^2 \sin \theta \cos \theta = 3$$

$$r^2 = \frac{3}{\sin \theta \cos \theta}$$

$$r = \pm \sqrt{\frac{3}{\sin \theta \cos \theta}}$$

D. $(x - 3)^2 + (y - 2)^2 = 13$

$$x^2 - 6x + 9 + y^2 - 4y + 4 = 13$$

$$r^2 - 6r \cos \theta + 9 - 4r \sin \theta + 4 = 13$$

$$r^2 - 6r \cos \theta - 4r \sin \theta = 0$$

$$17 \quad r(r - 6\cos \theta - 4\sin \theta) = 0$$

$r = 0$ or $r - 6\cos \theta - 4\sin \theta = 0$

↑
pole

$$\boxed{r = 4\sin \theta + 6\cos \theta}$$