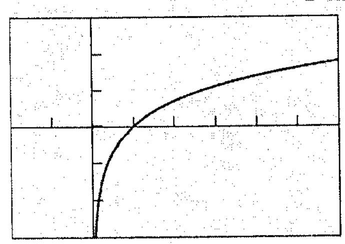
- Logarithmic functions are inverses of exponential functions.

BASIC FUNCTION The Natural Logarithmic Function



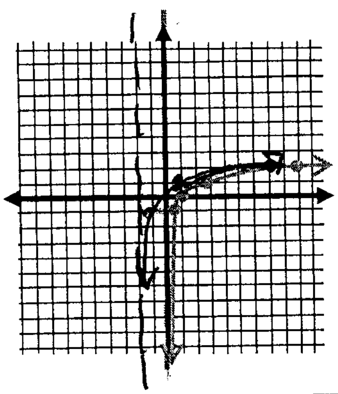
[−2, 6] by [−3, 3]

$f(x) = \ln x$
Domain: $(0, \infty)$
Range: All reals
Continuous on $(0, \infty)$
Increasing on $(0, \infty)$
No symmetry
Not bounded above or below
No local extrema
No horizontal asymptotes
Vertical asymptote: $x = 0$
End behavior: $\lim_{x \to \infty} \ln x = \infty$

Ex1) Describe how to transform the graph of $y = \ln x$ or $y = \log x$ into the graph of the given function. Then sketch the given function.

(a) $g(x) = \ln(x + 2)$

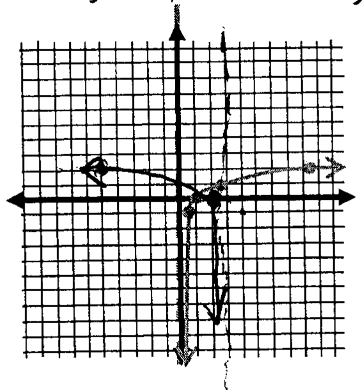
left 2



(b) $h(x) = \ln(3 - x)$

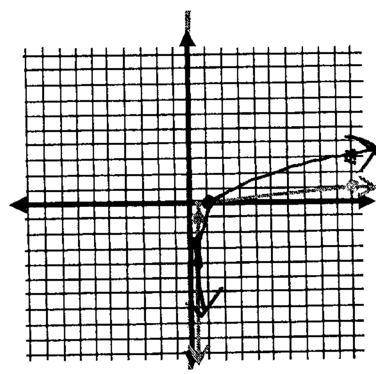
$\ln[-1(x-3)]$
$\ln(-x+3)$

right 3, reflect over y-axis



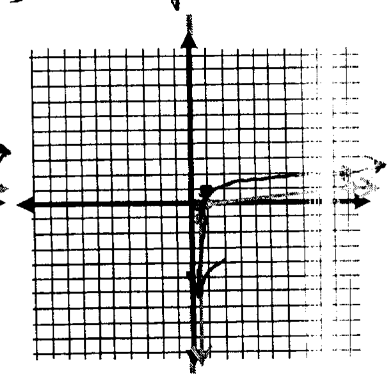
(c) $g(x) = 3 \log x$

vertical stretch *3



(d) $h(x) = 1 + \log x$

up 1



CHANGING BETWEEN EXPONENTIAL & LOGARITHMIC FORM

If $x > 0$, $b > 0$, & $b \neq 1$, then $y = \log_b x$ if and only if $x = b^y$

Ex2) Write each of the following in logarithmic or exponential form:

Log Form		**Exp Form**	**Exp Form**		**Log Form**
a) $\log_2 8 = 3$	→	$2^3 = 8$	e) $5^2 = 25$	→	$\log_5 25 = 2$
b) $\log_{27} 3 = ⅓$	→	$27^{\frac{1}{3}} = 3$	f) $9^{½} = 3$	→	$\log_9 3 = \frac{1}{2}$
c) $\log_{½} 16 = -4$	→	$\left(\frac{1}{2}\right)^{-4} = 16$	g) $(¼)^{-3} = 64$	→	$\log_{\frac{1}{4}} 64 = -3$
d) $\log_{25} 125 = \frac{3}{2}$	→	$25^{\frac{3}{2}} = 125$	h) $64^{-1/6} = ½$	→	$\log_{64} \frac{1}{2} = -\frac{1}{6}$

➢ Logarithms with base 10 are called Common logs & are written without a base.

➢ Logarithms with base e are called natural logs & are written with "LN" instead of log

Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y,

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

Ex3) Evaluate each of the following logs:

(a) $\log_5 125 = 3$ $5^x = 125$

(b) $\log_7 1 = 0$ $7^x = 1$

(c) $\log_9 9^4 = 4$ $9^x = 9^4$

(d) $11^{\log_{11} 7} = 7$

(e) $\log_8 32 = \frac{5}{3}$ $8^x = 32$ $(2^3)^x = 2^5$ $3x = 5$ $x = \frac{5}{3}$

(f) $\log_4 \frac{1}{64} = $ ‒

(g) $\log_3 \frac{1}{9} = -2$ $3^x = \frac{1}{9} = 3^{-2}$

(h) $\log_{\frac{1}{25}} 125 = -\frac{3}{2}$ $\left(\frac{1}{25}\right)^x = 125$ $5^{-2x} = 5^3$

When in this form $\log_b x$ ASK YOURSELF **"b to what power equals x"**

Ex4) Evaluate each of the following:

(a) $\log 100 = 2$ $10^x = 100$

(b) $\log \sqrt[5]{10} = \frac{1}{5}$ $10^x = \sqrt[5]{10}$

(c) $\log \frac{1}{1000} = -3$ $10^x = \frac{1}{1000}$

(d) $10^{\log 6} = 6$

Ex5) Solve the simple logarithmic equations below by changing them to exponential form:

(a) $\log x = 3$

$10^3 = x$

$x = 1000$

(b) $\log_2 x = 5$

$2^5 = x$

$x = 32$

Ex6) Evaluate each of the following:

(a) $\ln \sqrt{e} = \frac{1}{2}$ $e^x = \sqrt{e}$

(b) $\ln e^5 = 5$ $e^x = e^5$

(c) $e^{\ln 4} = 4$

Properties of Logarithms

Let b, R, and S be positive real numbers with $b \neq 1$

- **Product rule:** $\log_b (RS) = \log_b R + \log_b S$
- **Quotient rule:** $\log_b \frac{R}{S} = \log_b R - \log_b S$
- **Power rule:** $\log_b R^c = c \log_b R$

Change-of-Base Formula for Logarithms

For positive real numbers a, b, and x with $a \neq 1$ and $b \neq 1$,

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Ex7) Expand each of the following:

(a) $\log(8xy^4)$

(b) $\ln\left(\frac{\sqrt{x^2+5}}{x}\right)$

(a) $\log 8 + \log x + \log y^4 = \log 8 + \log x + 4\log y$

(b) $\ln(x^2+5)^{\frac{1}{2}} - \ln x = \frac{1}{2}\ln(x^2+5) - \ln x$

Ex8) Condense the following logarithmic expression: $\ln x^5 - 2\ln(xy) = \ln x^5 - \ln(xy)^2$

$\ln\left(\frac{x^5}{x^2y^2}\right) = \ln\left(\frac{x^3}{y^2}\right)$

Ex9) Given that $\ln 5 = a$ & $\ln 7 = b$ determine each of the following:

a) $\ln 35 = a+b$

$\ln(5 \cdot 7)$

$\ln 5 + \ln 7$

$a + b$

b) $\ln(5/7) = a-b$

$\ln 5 - \ln 7$

$a - b$

c) $\ln 175 = 2a+b$

$\ln(5 \cdot 5 \cdot 7)$

$\ln 5 + \ln 5 + \ln 7$

$2\ln 5 + \ln 7$

$2a + b$

d) $\log_5 7 = \frac{b}{a}$

$\frac{\ln 7}{\ln 5}$

$\frac{b}{a}$

e) $\log_7 35 = \frac{a+b}{b}$

$\frac{\ln 35}{\ln 7}$

$\frac{a+b}{b}$

f) $\log_5 175 = 2$

$\frac{\ln 175}{\ln 5}$

$\frac{2a+b}{a}$