Precalculus Unit 8 Notes—Sequences

Definition: A sequence is an ordered progression of numbers. This progression can be finite (meaning it ends), for example{3, 6, 9, 12, …, 21}. Or, it can be infinite, for example {3, 6, 9, 12, ….}.

Notation: a_n is used to denote a term in a sequence. The a alone actually has no value, however the n has a very significant meaning. It indicates the place of the term in the sequence being referred to.

can find any term

There are 2 ways to define these sequences explicitly & implicitly

have to know the previous term to get the next term

The **explicit definition** is like a formula.

Ex1) Find the first four terms of the given sequence.

a) $a_n = 2n+3$

5, 7, 9, 11 (a_1, a_2, a_3, a_4)

b) $a_n = 3\cdot 2^n$

6, 12, 24, 48 (a_1, a_2, a_3, a_4)

c) $a_n = n+\frac{1}{n}$

2, 2.5, $3\frac{1}{3}$, $4\frac{1}{4}$ (a_1, a_2, a_3, a_4)

d) $a_n = n^3+1$

2, 9, 28, 65 (a_1, a_2, a_3, a_4)

e) $a_n = 3-7n$

-4, -11, -18, -25 (a_1, a_2, a_3, a_4)

f) $a_n = (-2)^n$

-2, 4, -8, 16 (a_1, a_2, a_3, a_4)

The **recursive definition** has 2 parts:

(1) a term with which to begin
(2) a symbolic description of how the successive terms are related.

Ex2) Find the indicated terms of the given sequence.

a) $a_1 = 6$, $a_n = 4 + a_{n-1}$ ⇒ previous term

10, 14, 18, 22 (a_2, a_3, a_4, a_5)

b) $a_1 = 9$, $a_n = \frac{1}{3}\cdot a_{n-1}$

3, 1, $\frac{1}{3}$, $\frac{1}{9}$ (a_2, a_3, a_4, a_5)

c) $a_1 = 1, a_2 = 2$, $a_n = a_{n-1} + a_{n-2}$

2, 3, 5, 8 (a_2, a_3, a_4, a_5)

d) $a_1 = 4$, $a_n = 5\cdot a_{n-1}+2$

22, 112, 562, 2812 (a_2, a_3, a_4, a_5)

e) $a_1 = 1$, $a_n = \left(-\frac{1}{3}\right)^n \cdot a_{n-1}$

$\frac{1}{9}$, $-\frac{1}{243}$, $-\frac{1}{19683}$, $\frac{1}{4782969}$ (a_2, a_3, a_4, a_5)

f) $a_1 = 1, a_2 = 2$, $a_n = a_{n-1}\cdot a_{n-2}$

2, 2, 4, 8 (a_2, a_3, a_4, a_5)

Although it is possible to work with many different types of sequences, there are 2 that are most common.

arithmetic (where there is a common difference between each term) and geometric (where there is a common ratio between each pair of terms).

ARITHMETIC:
$a_n = a_1 + d(n-1)$ **, where** d **is the difference between each term (called the common difference)**

Ex3) State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the *n*th term of the sequence in terms of *n*.

a) 17, 21, 25, 29, …

$\underbrace{17, 21}_{+4}, \underbrace{21, 25}_{+4}, \underbrace{25, 29}_{+4}$ $d=4$

arithmetic

type: ______

$a_n = 17+4(n-1)$

b) 8, 12, 18, 27, …

$r=\frac{3}{2}$

type: geometric

$a_n = 8\left(\frac{3}{2}\right)^{n-1}$

c) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

type: neither

$a_n = \frac{n}{n+1}$

d) 11, 101, 1001, 10001, …

type: neither

$a_n = 10^n + 1$

e) 100, –50, 25, –12.5,…

$r=-\frac{1}{2}$

type: geometric

$a_n = 100\left(-\frac{1}{2}\right)^{n-1}$

f) 1, 4, 9, 16, …

type: neither

$a_n = n^2$

g) $\frac{2}{1}, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \ldots$

type: neither

$a_n = \frac{n+1}{n^2}$

h) 2a – 2b, 3a – b, 4a, 5a + b, …

$d=a+b$

type: arithmetic

$a_n = 2a-2b+(a+b)(n-1)$

Ex4) State whether each sequence is arithmetic, geometric, or neither. Then, find an explicit formula for the *n*th term of the sequence in terms of *n*.

a) $a_1 = 8,\ a_n = \frac{1}{2} \cdot a_{n-1}$

8, 4, 2, 1, …

type: geometric

$a_n = 8\left(\frac{1}{2}\right)^{n-1}$

b) $a_1 = 6,\ a_n = a_{n-1} + 10$

6, 16, 26, 36, …

type: arithmetic

$a_n = 6+10(n-1)$

c) $a_1 = \frac{1}{2},\ a_n = \frac{n}{n+1}(a_{n-1}+1)$

$\frac{1}{2}$, 1, 1.5, 2, 2.5, …

type: arithmetic

$a_n = \frac{1}{2}+.5(n-1)$

d) $a_1 = 1,\ a_n = a_{n-1} + 2n - 1$

1, 4, 9, 16, …

type: neither

$a_n = n^2$

e) $a_1 = 3,\ a_n = -2 \cdot a_{n-1}$

3, –6, 12, –24, …

type: geometric

$a_n = 3(-2)^{n-1}$

f) $2^{\frac{2}{3}}, 2^{\frac{5}{3}}, 2^{\frac{8}{3}}, \ldots$

$2^{\frac{2}{3}} \cdot 2^1 \quad 2^{\frac{5}{3}} \cdot 2^1$

type: geometric

$a_n = 2^{\frac{2}{3}}(2)^{n-1}$

Ex5) Find the indicated term of each arithmetic sequence: $d = -8$

a) $a_1 = 15$, $a_2 = 21$, $a_{20} = ?$

$d = 6$

$a_n = a_1 + d(n-1)$

$a_{20} = 15 + 6(20-1) = 129$

$\boxed{a_{20} = 129}$

b) $a_1 = 15$, $a_2 = 7$, $a_{20} = ?$

$a_{20} = 15 + -8(20-1)$

$\boxed{a_{20} = -137}$

Ex6) How many terms are in the finite arithmetic sequence

a) 18, 24, …, 336

$a_1 = 18$ $d = 6$

$a_n = 336$

$336 = 18 + 6(n-1)$

$318 = 6(n-1)$

$53 = n - 1$ $\boxed{n = 54}$

b) 178, 170, …, 2

$a_1 = 178$ $a_n = 2$ $d = -8$

$2 = 178 + -8(n-1)$

$-176 = -8(n-1)$

$22 = n - 1$ $\boxed{n = 23}$

Ex7) Find the number of multiples of 7 between 30 and 300.

$35, 42, \ldots, 294$

$294 = 35 + 7(n-1)$

$259 = 7(n-1)$

$37 = n - 1$ $\boxed{n = 38}$

Geometric Sequence: $a_n = a_1(r)^{n-1}$, where a_n is the nth term, r is the common ratio, & a_1 is the 1st term.

Ex8) Write an explicit representation of the pattern. Then find the 15th term.

$\frac{1}{243}, \frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \ldots$

$r = 3$

$a_n = \frac{1}{243}(3)^{n-1}$

$a_{15} = \frac{1}{243}(3)^{15-1} = \boxed{19683}$

Ex9) Given that $a_2 = 3$ & $a_5 = 24$ write an explicit formula if the sequence is a) arithmetic & b) geometric. Then find the values of a_3, and a_4 in each situation.

a) -4, 3, 10, 17, 24

$d = \frac{24-3}{3} = \frac{21}{3} = 7$

$a_n = -4 + 7(n-1)$

$a_3 = 10$ $a_4 = 17$

(These are called the Arithmetic means between a_2 & a_5)

b) $\frac{3}{2}$, 3, 6, 12, 24

$3 \cdot r^3 = 24$

$r^3 = 8$

$r = 2$

$a_n = \frac{3}{2}(2)^{n-1}$

$a_3 = 6$ $a_4 = 12$

(These are called the Geometric means between a_2 & a_5)