

There are 4 methods to solve quadratic equations algebraically:

- Factoring
- Extracting square roots
- Completing the square
- Using the quadratic formula

Example 1 Use factoring to solve each quadratic equation:

a) $5x^2 - 45 = 0$

$$5(x^2 - 9) = 0$$

$$5(x+3)(x-3) = 0$$

$$x+3=0 \quad x-3=0$$

$$x=-3 \quad x=3$$

c) $8x^2 - 2x - 18 = -15$

$$8x^2 - 2x - 3 = 0$$

$$(2x+1)(4x-3) = 0$$

$$2x+1=0 \quad 4x-3=0$$

$$2x=-1 \quad 4x=3$$

$$x = -\frac{1}{2} \quad x = \frac{3}{4}$$

b) $10x^2 + 13x - 3 = 0$

$$(5x-1)(2x+3) = 0$$

$$5x-1=0 \quad 2x+3=0$$

$$5x=1 \quad 2x=-3$$

$$x = \frac{1}{5} \quad x = -\frac{3}{2}$$

d) $6x^2 + 3x - 3 = 0$

$$3(2x^2 + x - 1) = 0$$

$$3(2x-1)(x+1) = 0$$

$$2x-1=0 \quad x+1=0$$

$$2x=1 \quad x=-1$$

$$x = \frac{1}{2}$$

Example 2 Use the square root method to solve each quadratic equation:

a) $\sqrt{(5x-1)^2} = \sqrt{12} \Rightarrow 5x-1 = \pm 2\sqrt{3}$

$$5x-1 = \pm 2\sqrt{3}$$

$$5x = 1 \pm 2\sqrt{3}$$

$$x = \frac{1 \pm 2\sqrt{3}}{5}$$

b) $\sqrt{(x+3)^2} = \sqrt{18} \Rightarrow x+3 = \pm 3\sqrt{2}$

$$x+3 = \pm 3\sqrt{2}$$

$$x = -3 \pm 3\sqrt{2}$$

c) $x^2 + 10x + 25 = 121$

$$\sqrt{(x+5)^2} = \sqrt{121}$$

$$x+5 = \pm 11$$

$$x = -5 \pm 11$$

$$x = 6 \quad x = -16$$

d) $x^2 + 6x + 9 = 8$

$$\sqrt{(x+3)^2} = \sqrt{8}$$

$$x+3 = \pm 2\sqrt{2}$$

$$x = -3 \pm 2\sqrt{2}$$

Example 3 Use completing the square to solve each quadratic equation:

a) $x^2 + 2x - 14 = 0$

$$x^2 + 2x + 1 - 14 - 1 = 0$$

$$\left(\frac{2}{2}\right)^2 = 1^2 = 1$$

$$(x+1)^2 - 15 = 0$$

$$\therefore (x+1)^2 = 15$$

$$x+1 = \pm \sqrt{15}$$

$$x = -1 \pm \sqrt{15}$$

c) $-x^2 - 2x + 5 = 0$

$$-1(x^2 + 2x + 1) + 5 + 1 = 0$$

$$\left(\frac{2}{2}\right)^2 = 1$$

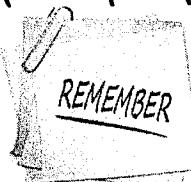
$$-(x+1)^2 + 6 = 0$$

$$-(x+1)^2 = -6$$

$$(x+1)^2 = 6$$

$$x+1 = \pm \sqrt{6}$$

$$x = -1 \pm \sqrt{6}$$



Quadratic Formula:

If $ax^2 + bx + c = 0$ where $a \neq 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm 3\sqrt{3}}{2}$$

Example 4 Use the quadratic formula to solve each quadratic equation:

a) $x^2 + 2x - 14 = 0$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-14)}}{2(1)}$$

$$\frac{-2 \pm \sqrt{60}}{2}$$

$$\frac{-2 \pm 2\sqrt{15}}{2} = -1 \pm \sqrt{15}$$

b) $2x^2 - 8x - 20 = 0$

$$2x^2 - 8x - 20 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-20)}}{2(2)}$$

$$\frac{8 \pm \sqrt{144}}{4}$$

$$\frac{8 \pm 4\sqrt{14}}{4} = 2 \pm \sqrt{14} \quad 9$$