There are 4 methods to solve quadratic equations algebraically:

- **Factoring**
- **Extracting square roots**
- **Completing the square**
- **Using the quadratic formula**

Example 1 Use factoring to solve each quadratic equation:

a) $5x^2 - 45 = 0$

$5(x^2-9)=0$

$5(x+3)(x-3)=0$

$x+3=0 \quad x-3=0$

$x=-3 \quad x=3$

b) $10x^2 + 13x = 3$

$10x^2+13x-3=0$

$(5x-1)(2x+3)=0$

$5x-1=0 \quad 2x+3=0$

$5x=1 \quad 2x=-3$

$x=\frac{1}{5} \quad x=-\frac{3}{2}$

c) $8x^2 - 2x - 18 = -15$

$8x^2-2x-3=0$

$(2x+1)(4x-3)=0$

$2x+1=0 \quad 4x-3=0$

$2x=-1 \quad 4x=3$

$x=-\frac{1}{2} \quad x=\frac{3}{4}$

d) $6x^2 + 3x - 3 = 0$

$3(2x^2+x-1)=0$

$3(2x-1)(x+1)=0$

$2x-1=0 \quad x+1=0$

$2x=1 \quad x=-1$

$x=\frac{1}{2}$

Example 2 Use the square root method to solve each quadratic equation:

a) $(5x-1)^2 = 12 \Rightarrow \sqrt{2\cdot 2\cdot 3}$

$5x-1=\pm 2\sqrt{3}$

$5x=1\pm 2\sqrt{3}$

$x=\frac{1\pm 2\sqrt{3}}{5}$

b) $(x+3)^2 = 18 \quad \sqrt{3\cdot 3\cdot 2}$

$x+3=\pm 3\sqrt{2}$

$x=-3\pm 3\sqrt{2}$

c) $x^2 + 10x + 25 = 121$

$(x+5)^2=\pm\sqrt{121}$

$x+5=\pm 11$

$x=-5\pm 11$

$x=6 \quad x=-16$

d) $x^2 + 6x + 9 = 8$

$(x+3)^2=\pm\sqrt{8}$

$x+3=\pm 2\sqrt{2}$

$x=-3\pm 2\sqrt{2}$

Example 3 Use completing the square to solve each quadratic equation:

a) $x^2+2x-14=0$

$x^2+2x+1-14-1=0$

$\left(\frac{2}{2}\right)^2=1^2=1$

$(x+1)^2-15=0$

$(x+1)^2=15$

$x+1=\pm\sqrt{15}$

$x=-1\pm\sqrt{15}$

b) $2x^2-8x-13=7$

$2x^2-8x-20=0$

$2(x^2-4x+4)-20-8=0$

$\left(\frac{-4}{2}\right)^2=4$

$2(x-2)^2-28=0$

$2(x-2)^2=28$

$(x-2)^2=14$

$x-2=\pm\sqrt{14}$ $x=2\pm\sqrt{14}$

c) $-x^2-2x+5=0$

$-1(x^2+2x+1)+5+1=0$

$\left(\frac{2}{2}\right)^2=1$

$-(x+1)^2+6=0$

$-(x+1)^2=-6$

$(x+1)^2=6$

$x+1=\pm\sqrt{6}$

$x=-1\pm\sqrt{6}$

d) $-2x^2+6x+9=0$

$-2(x^2-3x+\frac{9}{4})+9+\frac{9}{2}=0$

$\left(\frac{-3}{2}\right)^2=\frac{9}{4}$

$-2(x-\frac{3}{2})^2+\frac{27}{2}=0$

$-2(x-\frac{3}{2})^2=-\frac{27}{2}$

$(x-\frac{3}{2})^2=\frac{27}{4}$

$x-\frac{3}{2}=\pm\sqrt{\frac{27}{4}}$

$x=\frac{3}{2}\pm\frac{3\sqrt{3}}{2}$

$x=\frac{3\pm3\sqrt{3}}{2}$

REMEMBER

Quadratic Formula:

If $ax^2+bx+c=0$ where $a\neq0$ then

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example 4 Use the quadratic formula to solve each quadratic equation:

a) $x^2+2x-14=0$

$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-14)}}{2(1)}$

$\frac{-2\pm\sqrt{60}}{2}$

$\frac{-2\pm2\sqrt{15}}{2}=-1\pm\sqrt{15}$

b) $2x^2-8x-13=7$

$2x^2-8x-20=0$

$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(2)(-20)}}{2(2)}$

$\frac{8\pm\sqrt{224}}{4}$

$\frac{8\pm4\sqrt{14}}{4}=2\pm\sqrt{14}$