Precalculus Unit 3 Notes—Solving Rational Equations

**Steps to solve a rational equation**

1) Find the LCD.
2) Multiply every term on both sides of the equation by the LCD (objective is to divide out the denominators)
3) Solve for the variable.
4) Check for extraneous solutions!!! An extraneous solution is a solution to an equation that emerges from the process of solving the problem but is not a valid solution to the problem.

**Example 1** **Solve.**

$$x\left(x+\frac{3}{x}\right)=4\cdot x$$

$$(x)\left(x+\frac{3}{x}\right)=(4)(x)$$

$$x^2+3=4x$$

$$x^2-4x+3=0$$

$$(x-3)(x-1)=0$$

$$x-3=0 \quad x-1=0$$

$$\boxed{x=3} \quad \boxed{x=1}$$

STEP #1: Identify the LCD.

LCD = x

STEP #2: Multiply both sides by LCD.

(remember to DISTRIBUTE!!)

STEP #3: Solve.

STEP #4: Check your answers!!!!!

**Example 2** **Solve.**

A. $x+\dfrac{4}{x-4}=0$

$$(x-4)\left(x+\frac{4}{x-4}\right)=0(x-4)$$

$$x(x-4)+4=0$$

$$x^2-4x+4=0$$

$$(x-2)(x-2)=0$$

$$x-2=0$$

$$\boxed{x=2}$$

$$2+\frac{4}{2-4}\overset{?}{=}0$$

$$2+-2$$

B. $(x-3)(x-1)\left[\dfrac{2x}{x-1}+\dfrac{1}{x-3}\right]=\left[\dfrac{2}{x^2-4x+3}\right](x-3)(x-1)$

$(x-3)(x-1)$

LCD = $(x-3)(x-1)$

$$2x(x-3)+1(x-1)=2$$

$$2x^2-6x+x-1=2$$

$$2x^2-5x-3=0$$

$$(2x+1)(x-3)=0$$

$$2x+1=0 \quad x-3=0$$

$$\boxed{x=-\frac{1}{2}} \quad \cancel{x=3} \text{ extraneous}$$

$(2x+5)(3x-1)$ C. $\left[\frac{2x+2}{2x+5}-\frac{x-4}{3x-1}\right]=\left[\frac{5x^2+18}{6x^2+13x-5}\right](3x-1)(2x+5)$

$(3x-1)(2x+5)$

LCD: $(3x-1)(2x+5)$

$(3x-1)(2x+2)-(2x+5)(x-4)=5x^2+18$

$6x^2+4x-2-(2x^2-3x-20)=5x^2+18$

$4x^2+7x+18=5x^2+18$

$0=x^2-7x$

$0=x(x-7)$

$\boxed{x=0}$ $x-7=0$

$\boxed{x=7}$

D. $\frac{t+4}{t}+\frac{-4}{t-4}=\frac{-16}{t^2-4t}$

$t(t-4)$

LCD: $t(t-4)$

$(t+4)(t-4)+-4t=-16$

$t^2-16-4t=-16$

$t^2-4t=0$

$t(t-4)=0$

$\cancel{t=0}$ $t-4=0$

$\cancel{t=4}$

extraneous $\boxed{\text{no solution}}$

$(x-2)$ E. $\left(\frac{5x}{x-2}\right)=\left(7+\frac{10}{x-2}\right)(x-2)$

LCD: $x-2$

$5x=7(x-2)+10$

$5x=7x-14+10$

$-2x=-4$

$\cancel{x=2}$

$\boxed{\text{no solution}}$

$(x+1)(x-1)$ F. $\left[\frac{4x+1}{x+1}\right]=\left[\frac{12}{x^2-1}+3\right](x-1)(x+1)$

$(x-1)(x+1)$

LCD: $(x-1)(x+1)$

$(x-1)(4x+1)=12+3(x-1)(x+1)$

$4x^2-3x-1=12+3x^2-3$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x-5=0$ $x+2=0$

$\boxed{x=5}$ $\boxed{x=-2}$

G. $\frac{1}{x-2}+\frac{x-3}{7-x}=\frac{x+1}{-x^2+9x-14}$

$-1(x-7)-(x^2-9x+14)$

$-(x-7)(x-2)$

LCD: $-1(x-7)(x-2)$

$-1(x-7)(1)+(x-2)(x-3)=x+1$

$-x+7+x^2-5x+6=x+1$

$x^2-7x+12=0$

$(x-4)(x-3)=0$

$x-4=0$ $x-3=0$

$\boxed{x=4}$ $\boxed{x=3}$

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