

Notes (1.5) --- Transformations

A RIGID TRANSFORMATION - is a transformation that will leave the size and shape of a graph unchanged. This includes horizontal translations, vertical translations, reflections, or any combination of these.

A NON-RIGID TRANSFORMATION - is a transformation which will generally distort the shape of a graph. This includes horizontal or vertical stretches and shrinks.

Given a function $y = f(x)$

<p>$f(x) + k$ Vertical [Up] translation Ex: $f(x) = x^2 + 3$ <u>UP 3 units</u></p> <p>$f(x - h)$ Horizontal [Right] translation Ex: $f(x) = (x - 2)^2$ <u>Right 2 units</u></p> <p>$a \cdot f(x)$ if $a > 1$, <u>Vertical Stretch</u> ^{improper} Ex: $f(x) = 3(x - 2)^2$ <u>Stretch of 3</u></p> <p>$f(bx)$ if $b > 1$, Horizontal Shrink Ex: $f(x) = (2x)^2$ <u>Shrink of $\frac{1}{2}$</u></p> <p>$-f(x)$ <u>Reflect over x-axis</u> Ex: $f(x) = -(x)^2$ <u>Reflected over x-axis</u></p> <p>$f(x)$ <u>turns all negative y-values positive</u> Ex: $f(x) = (x)^2$ <u>turns all negative y-values positive</u></p>	<p>$f(x) - k$ Vertical [Down] Translation Ex: $f(x) = x^2 - 5$ <u>Down 5 units</u></p> <p>$f(x + h)$ Horizontal [Left] translation Ex: $f(x) = (x + 4)^2$ <u>Left 4 units</u></p> <p>if $a < 1$, <u>Vertical Shrink</u> <u>proper</u> Ex: $f(x) = \frac{1}{2}(x + 4)^2$ <u>Shrink of $\frac{1}{2}$</u></p> <p>if $b < 1$, Horizontal Stretch Ex: $f(x) = (\frac{1}{2}x)^2$ <u>Stretch of 2</u></p> <p>$f(-x)$ <u>reflect over y-axis</u> Ex: $f(x) = (-x)^2$ <u>Reflected over y-axis</u></p> <p>Ex: $f(x) = (x)^2$ <u>turns all negative y-values positive</u></p>
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NOTE: If there is a coefficient to x and a horizontal translation (a " b " and an " h ") then the coefficient should be factored out in order to truly see what the horizontal shift is.

$$y = a|b(x-h)| + k$$

EXAMPLE 1 Identifying Transformations

Describe how the graph of $y = |x|$ can be transformed to the graph of the given equation.

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|------------------------------------|------------------------------------|---|---|---|
| (a) $y = x - 4$
<u>Down 4</u> | (b) $y = x + 2 $
<u>left 2</u> | (c) $y = - x - 6 $
<u>Right 6</u>
<u>Reflect x-axis</u> | (d) $y = -x + 2 $ ^{GLP!}
<u>Reflect y-axis</u>
<u>Right 2</u> | (e) $y = - x + 3 - 7$
<u>Reflect x-axis</u>
<u>left 3</u>
<u>Down 7</u> |
|------------------------------------|------------------------------------|---|---|---|

EXAMPLE 2 Finding Equations of Transformations

Find an equation for the following transformations of the function $f(x) = \sqrt{x}$.

$$a\sqrt{b(x-h)} + k$$

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|---|--|--|
| (a) $f(x)$ is reflected over the y-axis and translated up 3 units | (b) $f(x)$ is vertically stretched by a factor of 3 and translated 4 units left. | (c) $f(x)$ is horizontally shrunk by a factor of $\frac{1}{2}$ & reflected over the x-axis |
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$$f(x) = \sqrt{-x} + 3$$

$$f(x) = 3\sqrt{x + 4}$$

$$f(x) = -\sqrt{\frac{1}{2}x}$$