

An identity is an equation that is true for every number in the domain of the equation.

### The Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

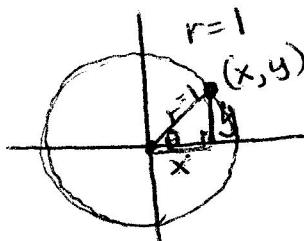
$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

### The Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$



$$\begin{aligned} x^2 + y^2 &= r^2 \\ (\cos \theta)^2 + (\sin \theta)^2 &= (1)^2 \\ \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} &= \frac{1}{\cos^2 \theta} \end{aligned}$$

### The Pythagorean Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta \quad 1 + \tan^2 \theta = \sec^2 \theta$$

### The Co-function Identities

Accomplished by reflecting across the y-axis and a phase shift right  $\frac{\pi}{2}$ .

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

**The Odd/Even Identities** Recall symmetry proofs from Unit 1. Even functions are defined when  $f(x) = f(-x)$  which shows symmetry about the y-axis. Odd functions are defined when  $f(-x) = -f(x)$  which shows symmetry about the origin.

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\cot(-\theta) = -\cot \theta$$

All the identities above (and others) are used to rewrite expressions in terms of predetermined trig functions.

### Examples of Rewriting Expressions:

A) Rewrite  $\cot \theta \cos \theta$  in terms of  $\sin \theta$

$$\frac{\cos \theta}{\sin \theta} \cdot \cos \theta = \frac{\cos^2 \theta}{\sin \theta} = \frac{1 - \sin^2 \theta}{\sin \theta}$$

B) Rewrite  $\frac{\cot \theta}{\cos \theta}$  in terms of  $\sin \theta$

$$\frac{\cos \theta}{\sin \theta} = \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} = \frac{1}{\sin \theta}$$

## Simplifying Expressions using Trigonometric substitutions and algebra:

You might want to try one or more of the following techniques:

- 1.) Change everything into sines and cosines.
- 2.) Use factoring to simplify the expression if possible.
- 3.) Get common denominators if there are fractions.
- 4.) Multiply both sides by a conjugate.
- 5.) Make substitutions using the identities.

\*\*\*When you simplify a trigonometric expression, the goal is to get a new expression which contains no fractions and contains the least number of terms possible.

Use fundamental identities, arithmetic, and/or algebraic properties to simplify the following expressions.

$$1) \sin\theta \cot\theta = \cancel{\sin\theta} \cdot \frac{\cos\theta}{\cancel{\sin\theta}} = \boxed{\cos\theta}$$

$$2) \cos^2\theta \csc\theta \sec\theta = \cos^2\theta \cdot \frac{1}{\sin\theta} \cdot \frac{1}{\cancel{\cos\theta}} = \frac{\cos\theta}{\sin\theta} = \boxed{\cot\theta}$$

$$3) \sin\theta \csc\theta \cot\theta = \cancel{\sin\theta} \cdot \frac{1}{\cancel{\sin\theta}} \cdot \frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta} = \boxed{\cot\theta}$$

$$4) \cos\theta(1 + \tan^2\theta) = \cos\theta \cdot \sec^2\theta = \cancel{\cos\theta} \cdot \frac{1}{\cancel{\cos^2\theta}} = \frac{1}{\cos\theta} = \boxed{\sec\theta}$$

$$5) \frac{1 - \cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \boxed{\tan^2\theta}$$

$$6) (\csc^2\theta - 1)(\sin^2\theta) = \cot^2\theta \cdot \sin^2\theta = \frac{\cos^2\theta}{\sin^2\theta} \cdot \sin^2\theta = \boxed{\cos^2\theta}$$

$$1) \sec(-x)\cos(-x) = \sec x \cdot \cos x = \frac{1}{\cos x} \cdot \cos x = \boxed{1}$$

$$8) \cot(-x)\cot\left(\frac{\pi}{2}-x\right) = -\cot x \cdot \tan x = -\frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x} = \boxed{-1}$$

$$9) \sin^3\theta + \sin\theta\cos^2\theta = \sin\theta (\sin^2\theta + \cos^2\theta) = \sin\theta \cdot 1 = \boxed{\sin\theta}$$

$$10) \sin\theta - \tan\theta\cos\theta + \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta - \frac{\sin\theta}{\cos\theta} \cdot \cos\theta + \sin\theta = \boxed{\sin\theta}$$

$$11) (1+\sin\theta)(1-\sin\theta) = 1 - \sin\theta + \sin\theta - \sin^2\theta = 1 - \sin^2\theta = \boxed{\cos^2\theta}$$

$$12) \cot^2\theta - \csc^2\theta = \csc^2\theta - 1 - \csc^2\theta = \boxed{-1}$$

$$\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta} = \frac{\cos^2\theta - 1}{\sin^2\theta} = \frac{-\sin^2\theta}{\sin^2\theta} = -1$$

$$13) \frac{1}{\cos^2 x} - \frac{1}{\cot^2 x} = \frac{1}{\cos^2 x} - \frac{1}{\frac{\cos^2 x}{\sin^2 x}} = \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = \boxed{1}$$

$$14) \frac{\cos x \cdot \cos x}{(1-\sin x)} - \frac{\sin x (1-\sin x)}{(\cos x)(1-\sin x)} \text{ LCD: } (1-\sin x) \cos x$$

$$\frac{\cos^2 x - \sin x + \sin^2 x}{\cos x (1-\sin x)} = \frac{1 - \sin x}{\cos x (1-\sin x)} = \frac{1}{\cos x} = \boxed{\sec x}$$