

Notes (2.3) - Polynomial Functions

**Graphing Polynomial Functions**

**Investigation 1 - End Behavior:**

$$x^4 + 1$$

1. Using a graphing calculator, graph the following functions:

- a.  $y = 3x^4 - 7x^3 + x^2 + 4$
- b.  $y = -1/2x^6 - x^5 + 2x^3 - 3x + 5$
- c.  $y = 2x^3 + 5x^2 - 3x - 2$
- d.  $y = -3x^5 + 7x^3 - 5$

2. What aspect of the polynomial affects the right end behavior?

$a \rightarrow y = 3x^4$  Degree [Highest]           
 ↑ pos or neg REB  $\rightarrow \infty$

3. What aspects of the polynomial affects the left end behavior?

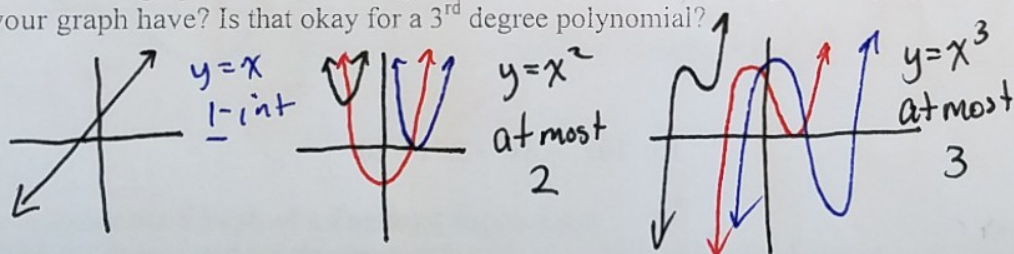
Highest Degree even  $\rightarrow \infty$   
 odd  $\rightarrow -\infty$

Coefficient positive even  $\rightarrow \infty$   
 negative even  $\rightarrow -\infty$   
 positive odd  $\rightarrow -\infty$   
 negative odd  $\rightarrow \infty$

Leading

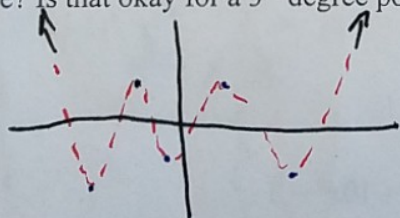
**Investigation 2 - Local Extrema and Zeros**

\* 1. Sketch a graph that has as many extrema as possible if the degree of a polynomial is 3. How many zeros does your graph have? Is that okay for a 3<sup>rd</sup> degree polynomial?



x-int  
 MAX # of zeros  
 $\leq$  Highest Degree

2. If the degree of a polynomial is 6, sketch a graph that has as many extrema as possible. How many zeros does your graph have? Is that okay for a 3<sup>rd</sup> degree polynomial?



MAX # of extrema  
 $\leq$  [Highest Degree - 1]

3. In general, if your degree is "n", how many extreme are possible? n-1

4. In general, if your degree is "n", how many zeros are possible? n

## Zeros of Polynomial Functions

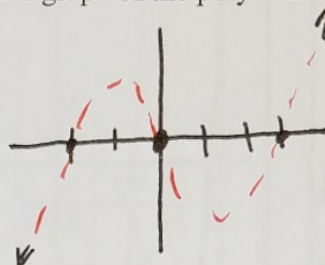
### Finding the Zeros of a Polynomial Function

Find the zeros of  $f(x) = x^3 - x^2 - 6x$  and then sketch the graph of the polynomial using your knowledge of intercepts and end behavior.

$$x(x^2 - x - 6)$$

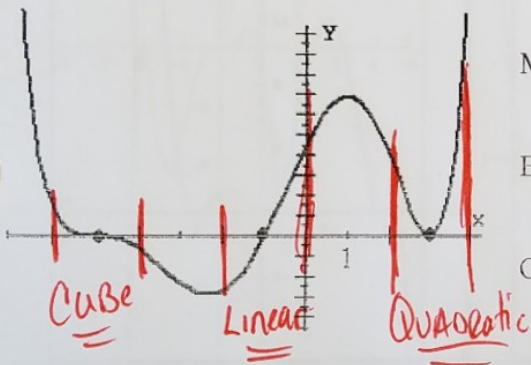
$$x(x-3)(x+2)$$

$$x=0 \quad x=3 \quad x=-2$$



**DEFINITION** Multiplicity of a Zero of a Polynomial Function

### Zeros of Odd and Even Multiplicity



**LINEAR**  
Multiplicity of 1:  $(x-2)^1$  power of factor is 1

**Quadratic**  
Even Multiplicity:  $(x-2)^{\text{EVEN}}$  power of factor even

**Cubic**  
Odd Multiplicity > 1:  $(x-2)^{\text{ODD} > 1}$  power of factor ODD AND Bigger than 1!

### Sketching the Graph of a Factored Polynomial

State the degree and list the zeros of the function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of  $f$  by hand.

