Logistic Growth Problems — Name: ______________________________

AP Calculus BC — Date: ______________________________

1	The carrying capacity for deer in a particular small town is 2,200, and the rate of increase in their numbers is proportional to both the number, n, of deer and $2{,}200 - n$. If there were 1,000 deer one month ago and 1,150 deer now, how many months will it take the deer to number 2,100?
2	*Guppy Population* A 2000-gallon tank can support no more than 150 guppies. Six guppies are introduced into the tank. Assume that the rate of growth of the population is $$\frac{dP}{dt} = 0.0015P(150 - P),$$ where time t is in weeks. **(a)** Find a formula for the guppy population in terms of t. **(b)** How long will it take for the guppy population to be 100? 125?
3	*Gorilla Population* A certain wild animal preserve can support no more than 250 lowland gorillas. Twenty-eight gorillas were known to be in the preserve in 1970. Assume that the rate of growth of the population is $$\frac{dP}{dt} = 0.0004P(250 - P),$$ where time t is in years. **(a)** Find a formula for the gorilla population in terms of t. **(b)** How long will it take for the gorilla population to reach the carrying capacity of the preserve?
4	Suppose that the growth of a population $y = y(t)$ is given by the logistic equation $$y = \frac{60}{5 + 7e^{-t}}$$ (a) What is the population at time $t = 0$? (b) What is the carrying capacity L? (c) What is the constant k? (d) When does the population reach half of the carrying capacity? (e) Find an initial-value problem whose solution is $y(t)$.
5	Suppose that the growth of a population $y = y(t)$ is given by the logistic equation $$y = \frac{1000}{1 + 999e^{-0.9t}}$$ (a) What is the population at time $t = 0$? (b) What is the carrying capacity L? (c) What is the constant k? (d) When does the population reach 75% of the carrying capacity? (e) Find an initial-value problem whose solution is $y(t)$.

6		Suppose that a population $y(t)$ grows in accordance with the logistic model $\frac{dy}{dt} = 10(1 - 0.1y)y$ (a) What is the carrying capacity? (b) What is the value of k? (c) For what value of y is the population growing most rapidly?

ANSWERS:

1. 10.820 years from now
2. a) p(t) = $\frac{150}{1 + 24e^{-.225t}}$ b) 17.205 weeks, 21.278 weeks
3. a) p(t) = $\frac{250}{1 + 7.9825e^{-.1t}}$ b) t → ∞
4. a) 5 b) 12 c) 1 d) 0.336 e) $\frac{dy}{dt} = \frac{7}{5}y\left(1 - \frac{y}{12}\right)$
5. a) 1 b) 1000 c) 0.9 d) 8.895 e) $\frac{dy}{dt} = .9y\left(1 - \frac{y}{1000}\right)$
6. a) 10 b) 10 c) 5