Applications of Integrals Review

1. Find the area under the curve $y = 2x - x^2$ from $x = 1$ to $x = 2$ with $n = 4$ left-endpoint rectangles.

2. Find the area under the curve $y = 2x - x^2$ from $x = 1$ to $x = 2$ using the Trapezoid Rule with $n = 4$.

3. Find the area under the curve $y = 2x - x^2$ from $x = 1$ to $x = 2$ using the Midpoint Formula with $n = 4$.

4. Find the area under the curve $y = 2x - x^2$ from $x = 1$ to $x = 2$.

5. Find the average value of $f(x) = 4x \cos x^2$ on the interval $\left[0, \sqrt{\frac{\pi}{2}}\right]$.

6. Find the average value of $f(x) = 2|x|$ on the interval $[-1, 1]$.

7. Find the length of the curve described by the parametric curve:
$x = \cos t$ and $y = \sin t$ from $t = \frac{\pi}{6}$ to $t = \frac{\pi}{3}$

8. Find the length of the curve described by $x = \frac{y^3}{18} + \frac{3}{2y}$ from y = 2 to y = 3.

9. Find the slope of the curve $r = 2\cos 4\theta$.

10. Find the slope of the curve $r = 2 - 3\sin\theta$ at $(2, \pi)$.

11. Find the area inside the limaçon $r = 4 + 2\cos\theta$.

12. Find the area inside $r = 2\cos\theta$ and outside $r = 1$.

13. Find the area inside the lemniscate $r^2 = 6\cos 2\theta$ and outside the circle $r = \sqrt{3}$.

Applications of Integrals Review

Free Response (You may use a calculator on these problems)

1. An object moving along a curve in the xy plane has its position given by <x(t), y(t)> at time t seconds, $0 \le t \le 1$, with $\frac{dx}{dt} = 8tcost$ units/sec and $\frac{dy}{dt} = 8tsint$ units/sec. At time t = 0, the object is located at (5, 11).
   a. Find the speed of the object at t = 1
   b. Find the length of the arc described by the curve's position from t = 0 to t = 1.
   c. Find the location of the object at t =π/2
   d. Find dy/dx & Find d^2y/dx
2. Two particles travel in the xy-plane. For time t≥0, the position of particle A is given by $x(t) = t + 1$ and $y(t) = (t+1)^2 - 2t - 2$, and the position of particle B is given by $x(t) = 4t - 2$ and $y(t) = -2t + 2$
   a. Find the velocity vector for each particle at time t = 2
   b. Set up an integral expression for the distance traveled by particle A from time t = 1 to t = 3. Do not evaluate the integral.
   c. At what time do the two particles collide? Justify your answer.
   d. Sketch the path of both particles from time t = 0 to t = 4. Indicate the direction of each particle along its path.

Answers.

1. 25/32	2. 21/32	3. 43/64	4. 2/3
5. $2\sqrt{2/\pi}$	6. 1	7. π/6	8. 47/36
9. $\frac{4sin4\theta sin\theta - cos4\theta cos\theta}{4sin4\theta cos\theta + cos4\theta sin\theta}$	10. 2/3	11. 18π	12. $\pi/3+\sqrt{3}/2$
13. $\frac{3\sqrt{3}-\pi}{2}$			

#1

a) 8
b) 4
c) (4π – 3, 19)
d) dy/dx = tan t $d^2y/dx = sec^3t/(8t)$

#2

a) Particle A <1, 2t> Particle B <4, -2>
b) $\int_1^3 \sqrt{1+4t^2}dt$
c) At t = 1
d)