AP Calculus AB

Midterm Exam Review Packet AB Calculus

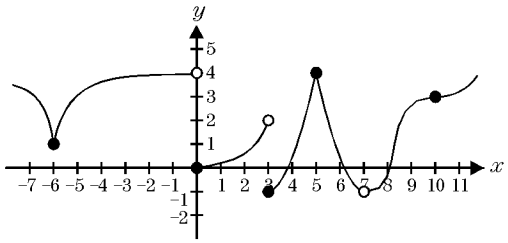
1. With respect to which of the following is the graph of $x^2 + xy + y^2 = 0$ symmetric?

   a) origin only
   b) y-axis only
   c) $y = x$ and origin
   d) x-axis only
   e) x-axis and y-axis

2. With respect to which of the following is the graph of $y = x^4 + 4x^2$ symmetric?

   a) x-axis only
   b) y-axis only
   c) origin only
   d) origin and y-axis
   e) x-axis and y-axis

This figure shows the graph of f. Use this figure to answer the following question(s).



3. $\lim_{x \to 7} f$ is

   a) 1 b) 2 c) -1 d) 4 e) 0

4. $\lim_{x \to 3} f$ is

   a) 1 b) 2 c) 3 d) -1
   e) no limit

5. $\lim_{x \to -6^-} f$ is

   a) 1 b) 2 c) -1 d) 4 e) 6

6. $\lim_{x \to 10^+} f$ is

   a) 1 b) 2 c) 3 d) -1
   e) no limit

7. At which of the following x-values is f continuous? Choose the BEST answer.

I. -6

II. 0

III. 3

IV. 5

V. 7

VI. 10

a) I, II, and IV
b) IV and VI
c) I, IV, and VI
d) II, III, and V
e) I and IV

8. $\lim_{x\to 0} \dfrac{\sqrt{x+9}-3}{x} =$

a) 0
b) $-\dfrac{1}{6}$
c) $\dfrac{1}{6}$
d) $\dfrac{1}{3}$
e) undefined

9. $\lim_{x\to 0} \dfrac{\sin 4x}{2x}$ is

a) -2
b) 0
c) $\frac{1}{4}$
d) 2
e) ∞

10. $\lim_{x\to -8^+} \dfrac{x+8}{|x+8|} =$

a) 1
b) does not exist
c) -1
d) ∞
e) ± 1

11. $\lim_{x\to\infty} \dfrac{x}{(x+2)(x-3)}$ is

a) 1
b) $-\infty$
c) $-\dfrac{2}{3}$
d) 0
e) ∞

12. Let f be defined as follows:

$$f(x) = \begin{cases} \dfrac{x^2-64}{x-8} & \text{for } x \neq 8, \\ 16 & \text{for } x = 8 \end{cases}$$

Which of the following are true about f?

I. $\lim_{x\to 8} f(x)$ exists

II. $f(8)$ exists

III. $f(x)$ is continuous at $x = 8$

a) None
b) I only
c) II only
d) I and II only
e) I, II, and III

13. The functions f and g are differentiable and have the values shown in the table.

If $A = f \cdot g$ then $A'(2) =$

a) 0 b) 53

c) -7 d) 3

e) 19

x	f	f'	g	g'
0	5	1	-7	$\frac{1}{4}$
2	8	3	-5	1
4	14	9	-3	4
6	26	27	-1	16

14. The functions f and g are differentiable and have the values shown in the table.

If $A = \left(\frac{f}{g}\right)$ then $A'(6) =$

a) -443

b) $\frac{443}{36}$

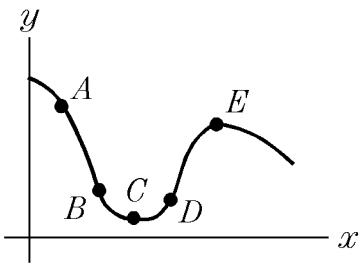
c) 443

d) $\frac{83}{9}$

e) $-\frac{443}{36}$

x	f	f'	g	g'
0	5	1	-7	$\frac{1}{4}$
2	8	3	-5	1
4	14	9	-3	4
6	26	27	-1	16

15. At which of the five points shown on the graph is $\frac{dy}{dx}$ positive? Choose the *best* answer.



a) A and E b) D only

c) C only d) C, D, and E

e) E only

16. At which of the five points shown on the graph is $\frac{d^2y}{dx^2}$ positive? Choose the *best* answer.

a) A and E b) B and D

c) C only d) B, C, and D

e) E only

17. Determine the interval where $f(x) = x^4 - 4x^3$ is decreasing.

a) $0 < x < \dfrac{1}{\sqrt{3}}$

b) $x < \dfrac{1}{3}$

c) $x < \dfrac{3}{\sqrt{3}}$

d) $x < \dfrac{1}{\sqrt{3}}$

e) $-\infty < x < 0$ and $0 < x < 3$

18. Let $f(x) = x^2 + 5x$ on $[0, 5]$

Which of the hypotheses of Rolle's Theorem are not satisfied on the given interval?

I. f is differentiable in (a, b)

II. f is continuous at $x = a$ and $x = b$

III. $f(a) = f(b) = 0$

IV. there is no c in (a, b) for which $f'(c) = 0$

a) III and IV b) IV only c) I and III

d) II and III e) III only

19. Given the function $f(x) = \dfrac{(x^2 + 4)}{x}$ satisfies the hypothesis of the Mean Value Theorem on the interval $[1, 4]$, find a number C in the interval $(1, 4)$ which satisfies this theorem.

a) 8 b) 2 c) 3 d) 16 e) 5

20. Which of the following functions does not satisfy the conditions of the Mean Value Theorem on $[1, 5]$?

a) $f(x) = |x| - 2$ b) $f(x) = \sqrt{x + 2}$

c) $f(x) = |x - 2|$ d) $f(x) = (x - 2)^2$

e) $f(x) = x^2 - 2x$

21. Find all points of inflection: $f(x) = x^3 - 12x$

a) $(0, 0)$, $(\pm\sqrt{12}, 0)$ b) $(0, 0)$

c) $(2, 0)$, $(-2, 0)$ d) $(2, -16)$, $(-2, 16)$

e) $(0, 0)$, $(2, -16)$

22. If $y = e^{x^2-3x}$, then $y' =$

a) $(2x-3)e^{x^2-3x}$ b) e^{x^2-3x}

c) $2x-3$ d) $(x^2-3x)e^{x^2-3x}$

e) $(2x-3)e^{2x-3}$

23. If $f(x) = \ln(\sin(3x-8))$, then $f'(x) =$

a) $\dfrac{\cos(3x-8)}{\sin(3x-8)}$ b) $\cos(\ln(3x-8))$

c) $3\cos(3x-8)$ d) $3\ln(\cos(3x-8))$

e) $3\cot(3x-8)$

24. Find $\dfrac{dy}{dx}$ for $y = 7^x$.

a) $x\ln 7$ b) $5x\ln 7$ c) $7\ln x$

d) $7^x \ln 7$ e) $7^x \ln x$

25. Find y' given $y^2e^{2x} + 3x = y^3$.

a) $\dfrac{2y^2e^{2x}+3}{3y^2-2ye^{2x}}$ b) $\dfrac{e^{2x}-3}{4}$

c) $\dfrac{ye^{2x}+3}{y-e^{2x}}$ d) $\dfrac{1-3y}{e^{2x}(3-2y)}$

e) $\dfrac{e^{2x}-3y}{e^{2x}-y^2}$

26. Which of the following gives the relative extrema for the function $f(x) = (x+1)^2(x-2)$?

a) Relative maximum: $x = -1$; Relative minimum: $x = 1$

b) Relative maxima: $x = 1$, $x = 3$; Relative minimum: $x = -1$

c) Relative minimum: $x = 2$

d) Relative maximum: $x = -1$; Relative minimum: $x = 2$

e) Relative maximum: $x = -1, 3$; Relative minimum: $x = 1$

27. Given $f(x) = \dfrac{x}{\tan x}$, find $f'\left(\dfrac{3\pi}{4}\right)$.

a) $\dfrac{3\pi}{4} - 1$ b) $\dfrac{3\pi}{2} - 1$ c) $\dfrac{-3\pi}{2} - 1$

d) $\dfrac{-3\pi}{4} - 1$ e) $\dfrac{-3\pi}{4} + 1$

28. Find the derivative, $\frac{dy}{dx}$, of $y = \frac{2x}{1-3x^2}$.

a) $-\frac{1}{3x}$ b) $-\frac{12x}{(1-3x^2)^2}$

c) $\frac{6x^2+2}{(1-3x^2)^2}$ d) $\frac{9x^2-2}{(1-3x)^2}$

e) $\frac{2x}{3(1-3x^2)^2}$

29. Find the slope of the tangent line to the graph of $f(x) = 2x(2x^2-1)$ at the point where $x = 1$.

a) $\frac{\sqrt{6}}{6}$ b) 2 c) $\sqrt{2}$ d) 10 e) 12

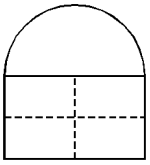
30. What is the slope of the tangent line to $x^4 + y^4 = 82$ at the point $(-1, 3)$?

a) $-\frac{1}{27}$ b) $\frac{1}{27}$ c) 27 d) -27 e) $\frac{1}{3}$

31. A window has a lower section in the shape of a rectangle and an upper portion in the shape of a semicircle surmounted on the upper side of this rectangle. The perimeter of the window is 15 units. Find the ratio of the radius of the circle to the height of the rectangle so that the area of the window is a maximum.

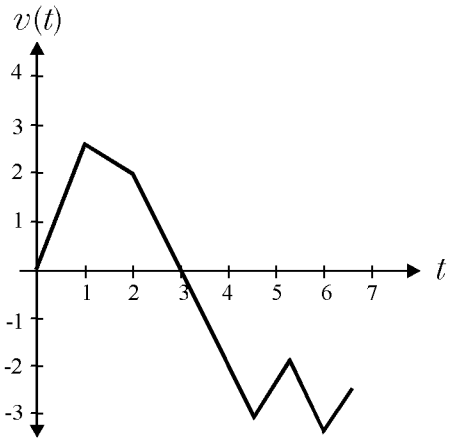
a) π b) $\frac{\pi}{2}$ c) $\frac{1}{2}$

d) 1 e) $\frac{\pi}{4}$



32. The graph shows the velocity of an object that is moving along a straight line for t on $[0, 6]$.

The object is furthest to the right when $t =$ _____.



a) 1 b) 3 c) 4 d) 6 e) 7

33. A projectile starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = t^3 - 6t^2 + 9t + 12$. What is the velocity of the particle at $t = 0$.

a) -9 b) 0 c) 6 d) 9 e) 12

34. The radius of a circle is increasing at the rate of 5 inches per minute. At what rate is the area increasing when the radius is 10 inches?

a) $100\pi\,\text{in}^2/\text{min}$ b) $50\pi\,\text{in}^2/\text{min}$

c) $5\pi\,\text{in}^2/\text{min}$ d) $500\,\text{in}^2/\text{min}$

e) $50\,\text{in}^2/\text{min}$

35. A ladder 15 ft in length leans against a vertical wall, with the bottom of the ladder 5 ft from the wall on a horizontal floor. If at that time the bottom end of the ladder is being pulled away at the rate of 2 ft/s, at what rate does the top of the ladder slip down the wall?

a) $\frac{\sqrt{2}}{2}$ ft/s b) 1 ft/s c) 3 ft/s

d) $\frac{2\sqrt{5}}{3}$ ft/s e) 2 ft/s

36. What is the domain of $f(x) = \dfrac{\sqrt{x^2 - 16}}{x + 3}$?

a) $x \neq 3$

b) $x \neq -3$

c) $|x| \geq 4$

d) $|x| \leq 4$ and $x \neq -3$

e) $x \geq 3$ and $x \neq -4$

37. Find the domain of $f(x) = \dfrac{1}{\sqrt{3 + 2x}}$.

a) $\left(-\infty, -\frac{3}{2}\right]$

b) $\left(-\infty, -\frac{3}{2}\right)$

c) $\left[-\frac{3}{2}, \infty\right)$

d) $\left(-\frac{3}{2}, \infty\right)$

e) $\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, \infty\right)$

1.
Answer: c

2.
Answer: b

3.
Answer: c

4.
Answer: e

5.
Answer: a

6.
Answer: c

7.
Answer: c

8.
Answer: c

9.
Answer: d

10.
Answer: a

11.
Answer: d

12.
Answer: e

13.
Answer: c

14.
Answer: a

15.
Answer: b

16.
Answer: d

17.
Answer: e

18.
Answer: a

19.
Answer: b

20.
Answer: c

21.
Answer: b

22.
Answer: a

23.
Answer: e

24.
Answer: d

25.
Answer: a

26.
Answer: a

27.
Answer: c

28.
Answer: c

29.
Answer: d

30.
Answer: b

31.
Answer: d

32.
Answer: b

33.
Answer: d

34.
Answer: a

35.
Answer: a

36.
Answer: c

37.
Answer: d