9/13/16

CALCULUS RECAP

$$m_{secant} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

DERIVATIVE

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

DERIVATIVE at a point $x=a$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$f(x) = \sqrt{x}$ at $x = a$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a}$$

$$f'(a) = \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} \cdot \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}}$$

$$f'(a) = \lim_{x \to a} \frac{x - a}{x - a(\sqrt{x} + \sqrt{a})}$$

$$f'(a) = \lim_{x \to a} \frac{1}{\sqrt{x} + \sqrt{a}} = \frac{1}{2\sqrt{a}}$$

Notation

$y = f(x)$ — y is a function of x

y' or $f'(x)$

$\frac{dy}{dx}$

$\frac{d}{dx} f(x)$

$D_x[y]$

} Derivative Notation

Average Rate of change

$f(x) = x^3 - x$ over $[1,3]$

$f(1) = 0$ $(1,0)$

$f(3) = 24$ $(3,24)$

$$\frac{f(3) - f(1)}{3 - 1} = 12$$

This means somewhere along interval $[1,3]$ your rate of change [slope] was equal to 12.

DRIVING TO VISIT OFFICER Morty.

You are leaving to visit your uncle officer Morty in Greenville [103 miles away]. You leave at 10:00am [CALLING MORTY TO LET HIM KNOW] You ARRIVE at his House at 11:15am and Uncle Morty Hands you a speeding ticket. Why?

AROC $\frac{f(1.25) - f(0)}{1.25 - 0} = \frac{103 - 0}{1.25}$

$= 82.4$ mph

OUCH!

UNCLE MORTY... YOU DON'T LOVE ME!

Where $f'(a)$ MIGHT FAIL TO EXIST.

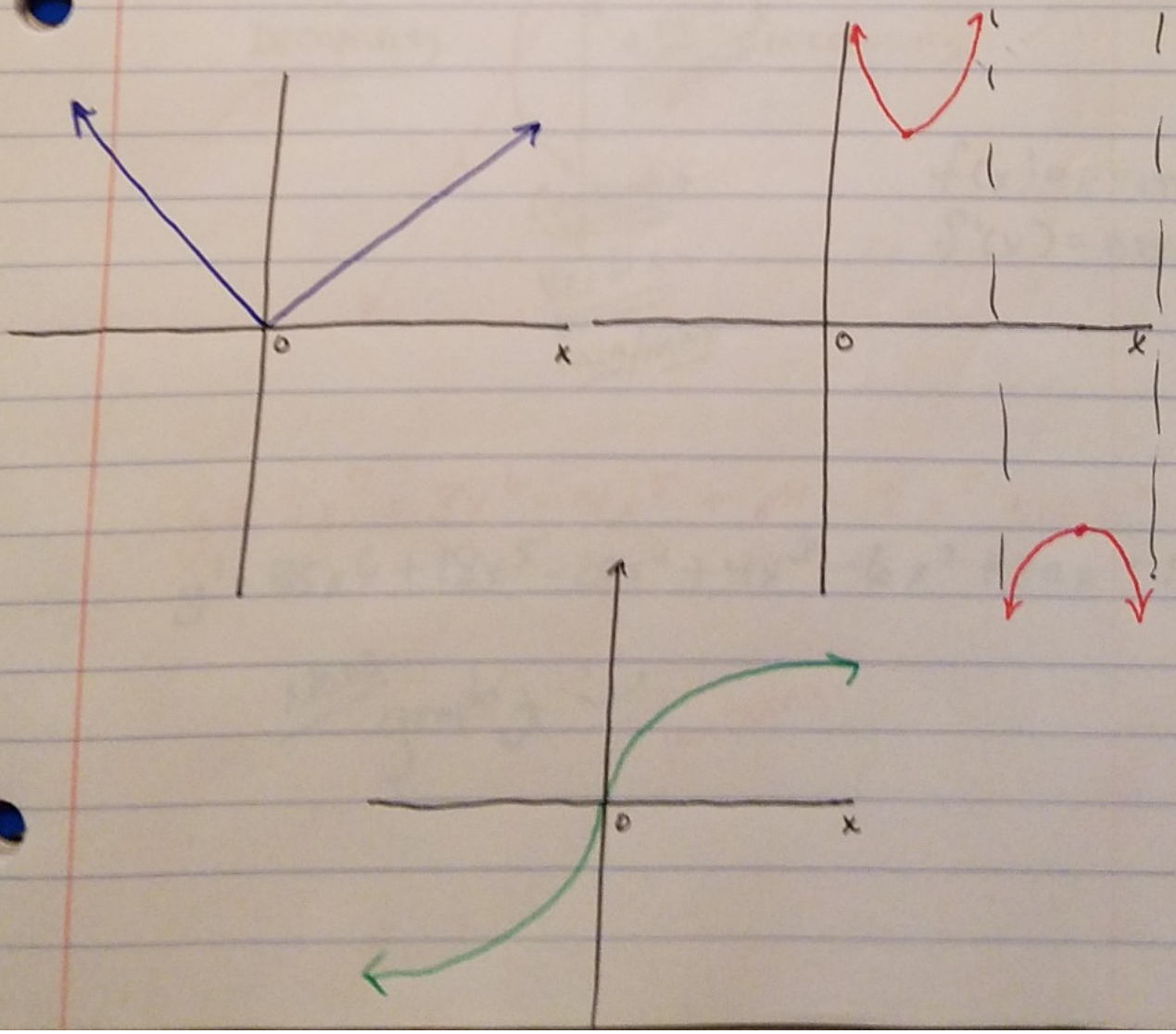
EPIC FAIL

① A corner – where one-sided derivatives differ.

② A cusp – slopes of secant lines approach ∞ & $-\infty$

③ A vertical TANGENT – slopes of secant lines approach either Both ∞ or Both $-\infty$

BOO!



1ST Derivative Rule

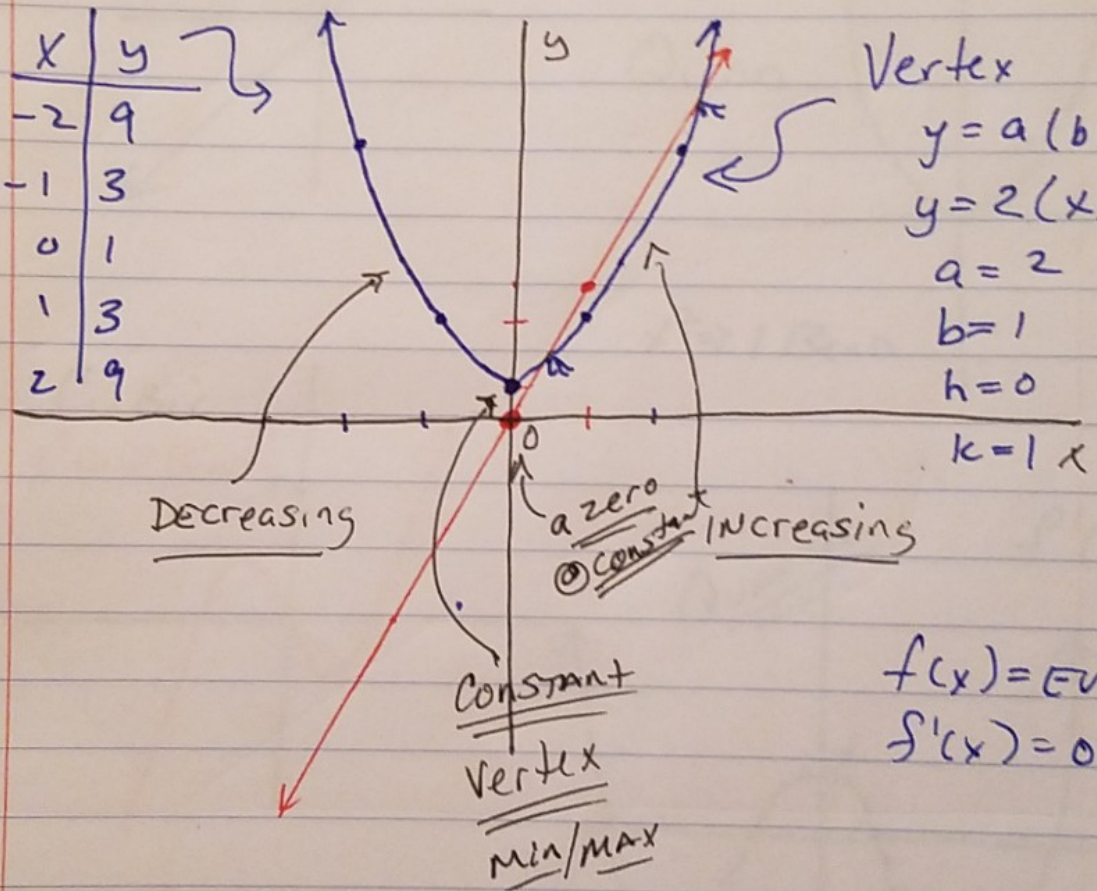
$f(x) = x^n$

$f'(x) = nx^{n-1}$

$f(x) = 2x^2 + 1$

$f'(x) = 4x$

x	y
-2	9
-1	3
0	1
1	3
2	9



Vertex

$y = a(b(x-h))^2 + k$

$y = 2(x)^2 + 1$

$a = 2$

$b = 1$

$h = 0$

$k = 1$

$f(x) = \text{EVEN}$

$f'(x) = \text{ODD}$

$y = 5x^7 + 3x^6 - 4x^5 + x^4 - 2x^3 + 10x^2 - 14x - 1$

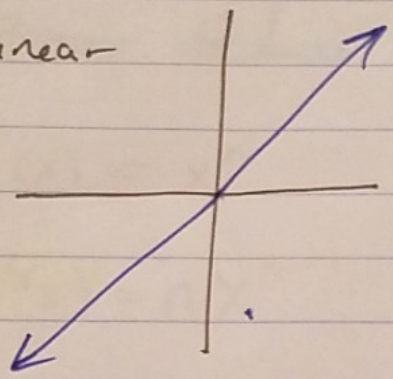
$y' = 35x^6 + 18x^5 - 20x^4 + 4x^3 - 6x^2 + 20x - 14 + 0$

Now graph it :)

kidding...

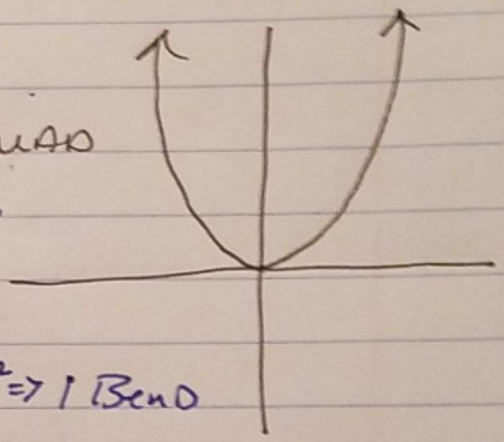
of Bends [MAX]

Linear



$x' \Rightarrow$ 0 Bends

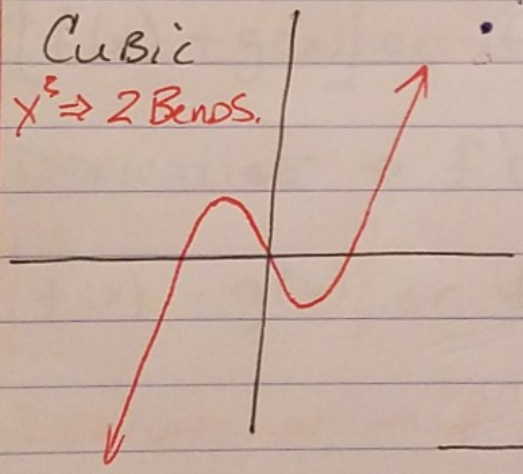
QUAD



$x^2 \Rightarrow$ 1 Bend

CUBIC

$x^3 \Rightarrow$ 2 Bends.



QUARTIC

$f'(x) =$ CUBIC

2 Bends