Unit #3 Notes – First & Second Derivative Tests

Match each term below with its correct definition.

Important Terms:

Critical Points	C
Absolute (Global) Maximum	I
Absolute (Global) Minimum	F
Relative (Local) Maximum	H
Relative (Local) Minimum	E
Function is Increasing	B
Function is Decreasing	J
Function is Concave Up	G
Function is Concave Down	D
Points of Inflection	A

Definitions:

A. function changes concavity/zeros of second derivative
B. derivative is positive
C. zeros of derivative or where derivative DNE
D. second derivative is negative
E. the point a graph changes from decreasing to increasing
F. minimum of entire interval
G. second derivative is positive
H. the point a graph changes from increasing to decreasing
I. maximum of entire interval
J. derivative is negative

THE FIRST DERIVATIVE TEST

Suppose that c is a critical number of a continuous function f:

- If f' changes from positive to negative at c, then f has a local maximum at c.
- If f' changes sign from negative to positive at c, then f has local minimum at c.
- If f' does not change signs at c, then f has no local extreme value at c.
- At left endpoint a - If $f' < 0$ for $x > a$, then f has a local maximum at a. If $f' > 0$ for $x > a$, then f has a local minimum at a.
- At right endpoint b - If $f' < 0$ for $x < b$, then f has a local minimum at b. If $f' > 0$ for $x < b$, then f has a local maximum at b.

SECOND DERIVATIVE ANALYSIS

- ☐ If $f'' > 0$ then the function is concave up
- ☐ If $f'' < 0$ then the function is concave down

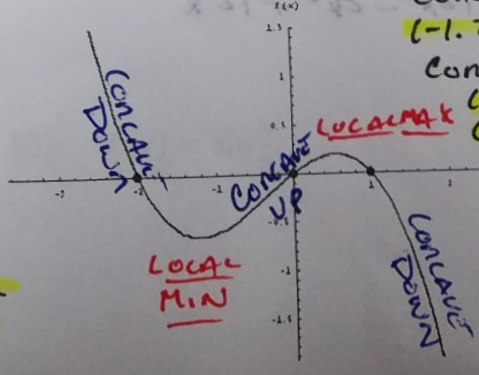
SECOND DERIVATIVE TEST

Suppose that c is a critical number of a continuous function f:

- ☐ If $f'(c) = 0$ and $f''(c)<0$, then f has a local max at $x = c$
- ☐ If $f'(c) = 0$ and $f''(c)>0$, then f has a local min at $x = c$
- ☐ If $f''(c) = 0$ of does not exist, the test fails. (When this happens, defer to the 1st derivative test.)

EX1) Given the graph of f', identify the following for f:

1. Maximums
2. Minimums
3. Intervals increasing/decreasing
4. Inflection points
5. Concavity



Concave Up
(-1.2, 0.5)
Concave Down
(-∞, -1.2)
(0.5, ∞)

P.O.I
x = -1.2
x = 0.5

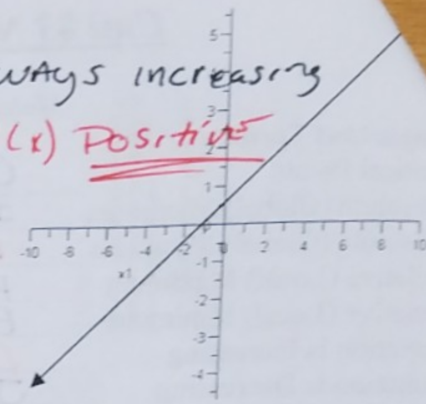
f(x) Inc [Above x-axis]
(-∞, -2) (0, 1)
MAX x = -2
x = 1
DEC [Below x-axis]
(-2, 0) (1, ∞)
Min x = 0

Ex2) Given the graph of f ', identify the following for f:

1. Maximums
2. Minimums
3. Intervals increasing/decreasing
4. Inflection points
5. Concavity

ALWAYS increasing

$f''(x)$ Positive



$f(x)$ DEC $(-\infty, -1)$

INC $(-1, \infty)$

MINIMUM @ $x = -1$

CONCAVE UP

$(-\infty, \infty)$

NO P.O.I

NO MAX

Ex3) Find all extrema on the given intervals:

a) $f(x) = x^3 - 6x + 5$ $[-2, 3]$

$f'(x) = 3x^2 - 6$

b) $f(x) = 3x^{2/3}$ $[-1, 2]$

$f'(x) = \frac{2}{\sqrt[3]{x}}$

c) $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$ $(-\infty, \infty)$

$f'(x) = \dfrac{x}{(4 - x^2)^{3/2}}$

Ex4) Find all the intervals where $f(x) = 4x^3 - 3x^2 - 18x + 6$ is increasing & all intervals where f is decreasing.

$f'(x) = 12x^2 - 6x - 18$

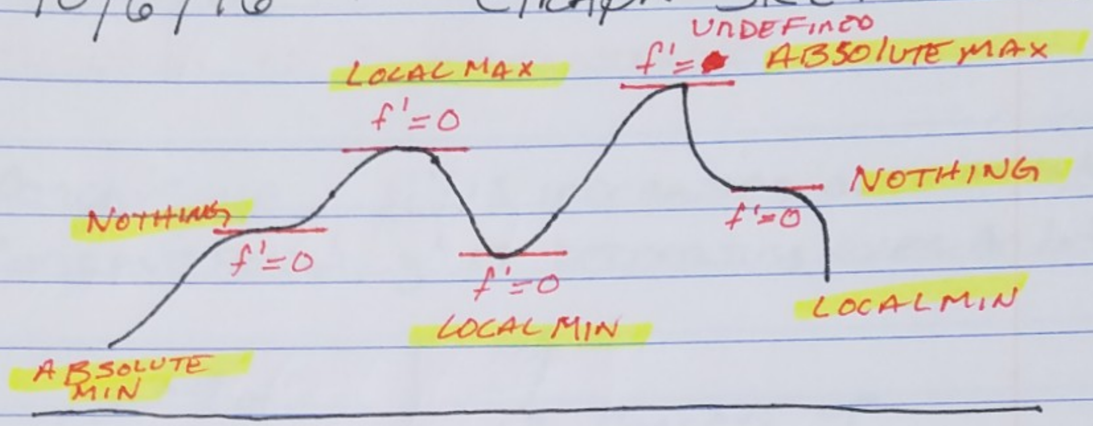
Ex5) Find the relative extrema of $y = \sin(x) - 2\cos(x)$ in the interval $[0, 2\pi]$. Justify your answer. CALCULATOR

$y' = \cos x + 2\sin x$

Ex6) Find the points of inflection for $g(x) = 3x^4 - 8x^3 + 6x^2$. Justify your answer.

$g'(x) = 12x^3 - 24x^2 + 12x$

10/6/16 Graph SKETCHING



FIRST DERIVATIVE TEST [RISES AND FALLS]
f IS CONTINUOUS

IF f' CHANGES from (+) to (−) @ c,
THEN f HAS A LOCAL MAXIMUM @ c

IF f' CHANGES from (−) to (+) @ c,
THEN f HAS A LOCAL MINIMUM @ c.

IF f' DOES NOT CHANGE Sign @ c,
THEN f DOES NOT HAS AN EXTREME VALUE @ c

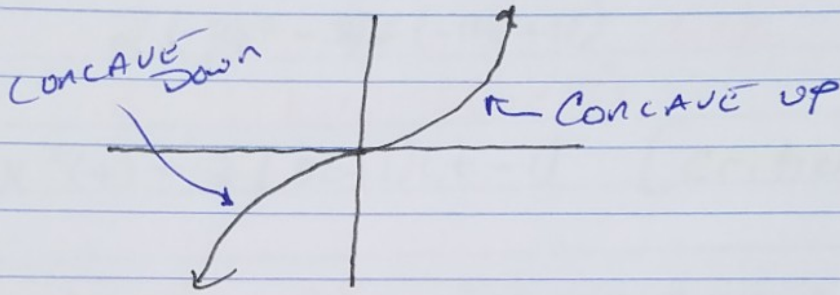
@ AN ENDPOINT
IF $f'<0$ or $f'>0$ for $\forall\ x>a$, THEN f has
A LOCAL MAXIMUM / MINUMIM @ a

CONCAVITY

function is Differentiable

CONCAVE UP y' is increasing OVER An Interval

CONCAVE DOWN y' is DECREASING OVER An Interval



f is CONCAVE UP if $y'' > 0$

f is CONCAVE DOWN if $y'' < 0$

POINT OF INFLECTION [POI] NO NOT A HAWAIIAN FOOD

THIS OCCURS WHEN CONCAVITY CHANGES.

SECOND DERIVATIVE TEST ~~FOLA~~

if $f'(c) = 0$ AND $f''(c) > 0$ LOCAL MINIMUM

if $f'(c) = 0$ AND $f''(c) < 0$ LOCAL MAXIMUM

EXAMPLE $x(t) = 2t^3 - 14t^2 + 22t - 5,\ t \geq 0$

$x'(t) = 6t^2 - 28t + 22$ FACTOR IT

$2(\underbrace{3t^2 - 14t + 11}_{33})$

$-3 \quad -11$

$2((3t^2 - 3t) + (-11t + 11))$ GCF

$2(3t(t-1) - 11(t-1))$

$x'(t) = 2(3t-11)(t-1)$ [CRITICAL POINTS]

$x''(t) = 12t - 28$ FACTOR AGAIN

$4(3t-7)$

$x''(t) = 4(3t-7)$ [CONCAVITY]

$x'(t) = 2(3t-11)(t-1)$

$0 = 2(3t-11)(t-1)$

$3t-11 = 0 \qquad t-1 = 0$

$t = \frac{11}{3} \qquad t = 1$

$x''(t) = 4(3t-7)$

$0 = 4(3t-7)$

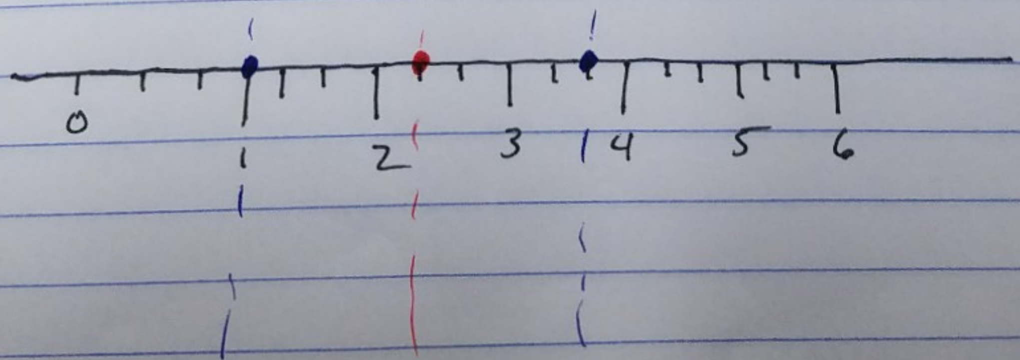
$3t - 7 = 0$

$t = \frac{7}{3}$

$-$ (DEC) CONCAVE DOWN | $+$ (INC) CONCAVE UP

$+$ (INC) | $-$ (DEC) | $+$ (INC)

TEST POINTS of $x'(t)$ $x''(t)$



EX. 3 Find all extrema

$f(x) = x^3 - 6x + 5 \quad [-2, 3]$

$f'(x) = 3x^2 - 6$

$0 = 3x^2 - 6$

$6 = 3x^2$

$2 = x^2$

$\pm\sqrt{2} = x$

$+ \mid - \mid + \quad f'(x)$

$-2 \quad -\sqrt{2} \quad 0 \quad \sqrt{2} \quad 3$

$(-\sqrt{2}, -0.657)$ ABS MIN

$(3, 14)$ ABS MAX

x	$f(x)$
-2	9
$-\sqrt{2}$	$(-\sqrt{2})^3 - 6(-\sqrt{2}) + 5 = -0.657$
$\sqrt{2}$	$(\sqrt{2})^3 - 6(\sqrt{2}) + 5 = 10.657$
3	14

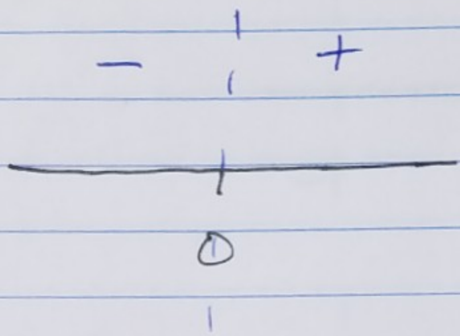
EXAMPLE $f(x) = 3x^{\frac{2}{3}}$ $[-1, 2]$

$f'(x) = \dfrac{2}{3\sqrt{x}}$ $f'(x)$ IS UNDEFINED $x=0$

CAN'T DIVIDE BY 0.

$0 = \dfrac{2}{\sqrt[3]{x}}$

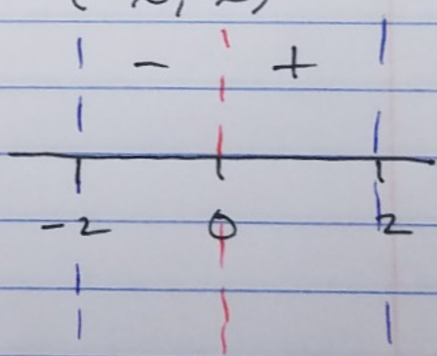
NO WORKIE



x	$f(x)$
-1	3
0	0 ABS. MIN $(0,0)$
2	$3\sqrt[3]{2^2} = 4.762$ $(2, 4.762)$ ABS. MAX.

EXAMPLE $f(x) = \dfrac{1}{\sqrt{4-x^2}}$ $(-\infty, \infty)$

$f'(x) = \dfrac{x}{(4-x^2)^{3/2}}$



$0 = \dfrac{x}{(4-x^2)^{3/2}}$

$f'(x) = 0$ @ $x = 0$

$f'(x)$ UNDEFINED $x = \pm 2$

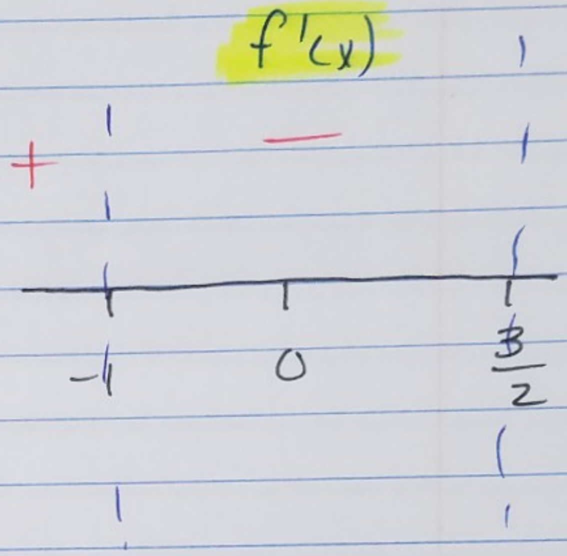
ABSOLUTE MIN $(0, \frac{1}{2})$

EXAmple: $f(x) = 4x^3 - 3x^2 - 18x + 6$

$f'(x) = 12x^2 - 6x - 18$

$0 = 12x^2 - 6x - 18$

$0 = 6(2x^2 - x - 3)$



$\underbrace{2x^2 - x - 3}_{-6}$

$(2x^2 + 2x) + (-3x - 3)$

$(2x-3)(x+1)$

$0 = 6(2x-3)(x+1)$

$0 = 2x - 3$ $\quad$ $x + 1 = 0$

$\frac{3}{2} = x$ $\quad$ $x = -1$

Increasing $(-\infty, -1)$

$(\frac{3}{2}, \infty)$

Decreasing $(-1, \frac{3}{2})$