

W/S Practice 4.3 Parametric FRQs (solns by Borchert)

① $x(4) = x(2) + \int_2^4 3 + \cos(t^2) dt$
 $x(4) = x(2) + \int_2^4 3 + \cos(t^2) dt$
 $x(4) = 1 + \int_2^4 3 + \cos(t^2) dt \approx 7.133$

b) $\frac{dy}{dt} = -7$ at $(x(2), y(2)) = (1, 8)$
 $\frac{dx}{dt} = 3 + \cos(t^2)$
 $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-7}{3 + \cos(4)}$ at $t=2 \approx -2.983$
 $y - 8 \approx -2.983(x - 1)$

c) $= \sqrt{[x'(2)]^2 + [y'(2)]^2}$
 $= \sqrt{(3 + \cos(4))^2 + (-7)^2} \approx 7.383$

d) $(x(t), y(t))$ $\frac{dy}{dx} = 2t + 1$ acceleration vector $t=4$

$\frac{dx}{dt} = 2t \cos(t^2)$ $x''(4) = 24.814$ $y''(4) = 24.814$
 $\frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{dy}{dt}$
 $-2t \sin(t^2) \cdot \frac{dy}{dt} (2t+1) (3 + \cos(t^2)) =$
 $(2.303, 24.814) (2t+1) (-2t \sin(t^2)) + 2(3 + \cos(t^2))$
 24.814

② $\frac{dx}{dt} = \sqrt{t^4 + 9}$ $\frac{dy}{dt} = 2e^t + 5e^{-t}$ @ $t=0$ point $(4, 1)$

a) speed $= \sqrt{[x'(0)]^2 + [y'(0)]^2}$
 $= \sqrt{(3)^2 + (7)^2} = \sqrt{58}$

acceleration vector $\langle x''(0), y''(0) \rangle$
 $\langle \frac{1}{2}(t^4+9)^{-1/2}(4t^3), 2e^t - 5e^{-t} \rangle$
 $\langle 0, -3 \rangle$

b) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2e^t + 5e^{-t}}{\sqrt{t^4 + 9}}$ at $t=0$ $\frac{dy}{dx} = \frac{7}{3}$
 $y - 1 = \frac{7}{3}(x - 4)$

c) $\int_0^3 \sqrt{(\sqrt{t^4+9})^2 + (2e^t+5e^{-t})^2} dt \approx 45.227$

d) $x(3) = x(0) + \int_0^3 \sqrt{t^4+9} dt$
 $\approx 4 + 13.931 \approx 17.931$

③ $(x(t), y(t)) \quad t \geq 0 \quad \frac{dx}{dt} = 12t - 3t^2 \quad \frac{dy}{dt} = \ln(1 + (t-4)^4)$

a) Find acceleration vector at $t=2$ and speed $t=2$

acceleration vector $\langle x''(2), y''(2) \rangle$
 $\langle 12 - 6t, \frac{4(t-4)^3}{1+(t-4)^4} \rangle$
 $\langle 0, -\frac{32}{17} \rangle$ or $\langle 0, -1.882 \rangle$

speed $\sqrt{(v_x(2) - 3(2))^2 + [\ln(1+(2-4)^4)]^2}$
 $\approx \boxed{2.3299}$

b) $y(t) = y(0) + \int_0^t \ln(1+(t-4)^4) dt$

$y(2) = 5 + \int_0^2 \ln(1+(t-4)^4) dt = 13.671$

c) At $t=2$ slope = $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\ln(17)}{12} = .236$

$y - 13.671 = .236(x - 3)$ point $(3, 13.671)$

d) $x'(t)$ and $y'(t) = 0$ at $t = 4$

$\frac{dx}{dt} = 12t - 3t^2 = 0 \quad \frac{dy}{dt} = \ln(1+(t-4)^4) = 0$
 $3t(4-t) = 0 \quad 1+(t-4)^4 = 1$
 $t=0 \quad t=4 \quad (t-4)^4 = 0$
 $t=4$

① $t=1 \quad (2, -3)$

a) $\frac{dx}{dt} = \tan(e^{-t}) \quad \frac{dy}{dt} = \sec(e^{-t})$

pt. $\frac{dy}{dx}$ at $(2, -3)$
 st. eqn. of line $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec(e^{-t})}{\tan(e^{-t})} = \frac{1}{\cos(e^{-t}) \sin(e^{-t})} = \frac{1}{\sin(e^{-t})}$

$\left. \frac{dy}{dx} \right|_{(2, -3)} = \frac{1}{\sin(e^{-1})} \approx 2.781 \quad y + 3 \approx 2.781(x - 2)$

pt. vector b) acceleration vector $\langle x''(1), y''(1) \rangle$

$\langle -.423, -.152 \rangle$

speed $= \sqrt{(x''(1))^2 + (y''(1))^2} = \sqrt{(\sec(e^{-1}))^2 + (\tan(e^{-1}))^2} \approx \boxed{1.139}$

pt. integral pt. answer distance = $\int_1^2 \sqrt{(x'(t))^2 + (y'(t))^2} dt \approx \boxed{1.059}$

pt. $x(0)$ expression d) $x(0) = x(1) - \int_0^1 x'(t) dt = 2 - .77553 > 0$

pt. $x'(t) > 0$ pt. conclusion \rightarrow reason The particle starts on the positive side of the y-axis. Since $x'(t) > 0$ for all $t \geq 0$ the object is always moving right and therefore never on the y-axis.

2006 BC3

⑤ $\frac{dy}{dt} = \sin^{-1}(1-2e^{-t})$ $\frac{dy}{dt} = \frac{4t}{1+t^3}$ $t=2$ (6, -3)

lpt. vector
lpt. speed

a) acceleration vector $\langle x''(2), y''(2) \rangle$
 $\langle -3.96, -.741 \rangle$

speed $\sqrt{(x'(2))^2 + (y'(2))^2} \approx \boxed{1.208}$

lpt. x(t)=0
lpt. answer

b) $\frac{dx}{dt} = 0$ $\sin^{-1}(1-2e^{-t}) = 0$
 $1-2e^{-t} = 0$
 $-2e^{-t} = -1$
 $e^{-t} = 1/2$
 $-t = \ln(1/2)$
 $t = \ln(2) = \boxed{.693}$ $\frac{dy}{dt} \neq 0$ when $t = \ln(2)$

lpt. m(t)

c) $m(t) = \frac{4t}{1+t^3} \cdot \frac{1}{\sin^{-1}(1-2e^{-t})}$

lpt. limit
value

$\lim_{t \rightarrow \infty} = \frac{4t}{1+t^3} \cdot \frac{1}{\sin^{-1}(1-2e^{-t})}$
 $= 0 \cdot \frac{1}{\sin^{-1}(1)} = 0$

lpt. integrand

d) $y = C$

lpt. limits

lpt. initial
value

consistent
with
limit

$C = \lim_{t \rightarrow \infty} y(t) = -3 + \int_2^{\infty} \frac{4t}{1+t^3} dt$

2007 BC2
Form B

lpt. ⑥ a) speed at $t=4$ $t=0$ (-3, -4) $\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right)$ $\frac{dy}{dt} = \ln(t^2+1)$
 $\sqrt{(x'(4))^2 + (y'(4))^2} \approx \boxed{2.912}$

lpt. integral
lpt. answer

b) Total distance from $0 \leq t \leq 4$
 Distance = $\int_0^4 \sqrt{(x'(t))^2 + (y'(t))^2} dt = \boxed{6.423}$

lpt. integrand
lpt. x(0)=-3
lpt. answer

c) $x(4) = x(0) + \int_0^4 \arctan\left(\frac{t}{1+t}\right) dt \approx$
 $x(4) = -3 + 2.107941223 \approx \boxed{-.892}$

lpt. $\frac{dy}{dx} = 2$
at

lpt. t value

lpt. values
for x'' and y''

d) slope = 2 $\frac{dy}{dx} = 2$ $\ln(t^2+1) = 2 \tan^{-1}\left(\frac{t}{1+t}\right)$
 $t \approx \boxed{1.3576631}$
 acceleration vector
 plugged in calc. $\rightarrow \langle x''(1.3576631), y''(1.3576631) \rangle$
 $\langle .135, .955 \rangle$

⑦ $t=4$ (1,5) $\frac{dx}{dt} = \sqrt{3}t$ $\frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right)$

a) acceleration vector at $t=4$

1 pt.
answer

$\langle x''(4), y''(4) \rangle$ plugged in calc.

$\langle .433, -11.872 \rangle$

1 pt. integrand
(pt. uses $y(4)$)
1 pt. answer

b) $y(0) = y(4) - \int_0^4 3\cos\left(\frac{t^2}{2}\right) dt$

$y(0) = 5 - 3.399395876 \approx \boxed{1.6006}$

1 pt.
expression
for
speed
1 pt. equation
1 pt. answer

c) $\sqrt{(x'(t))^2 + (y'(t))^2} = 3.5$ plugged in $\frac{y}{x}$ found
intersection

$t \approx \boxed{2.2256}$

1 pt. integral
1 pt. answer

d) distance = $\int_0^4 \sqrt{(x'(t))^2 + (y'(t))^2} dt \approx \boxed{13.182}$