10/31/2016 HAPPY HALLOWEEN

{ THE FUNDAMENTAL THEOREM OF CALCULUS. }

If f is continuous at every point of $[a,b]$, and if F is any antiderivative of f on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

THIS IS KNOWN AS THE INTEGRAL EVALUATION THEOREM.

EXAMPLE: $\int_{-1}^{3} (x^3+1)\,dx$

$$\int_{-1}^{3} (x^3+1)\,dx = \left[\frac{x^4}{4} + x\right]_{-1}^{3}$$

WAIT WHAT ABOUT "C"

OK!! $\left[\frac{x^4}{4} + x + C\right]_{-1}^{3}$ [NOW WATCH]

PLUGIN 3 — PLUG IN −1

$$\left[\frac{x^4}{4} + x + C\right] - \left[\frac{x^4}{4} + x + C\right]$$

$$\left(\frac{81}{4} + 3 + c\right) - \left(\frac{1}{4} - 1 + c\right)$$

$$\left(\frac{81}{4} + \frac{12}{4} + c\right) - \left(\frac{1}{4} - \frac{4}{4} + c\right)$$ DISTRIBUTE

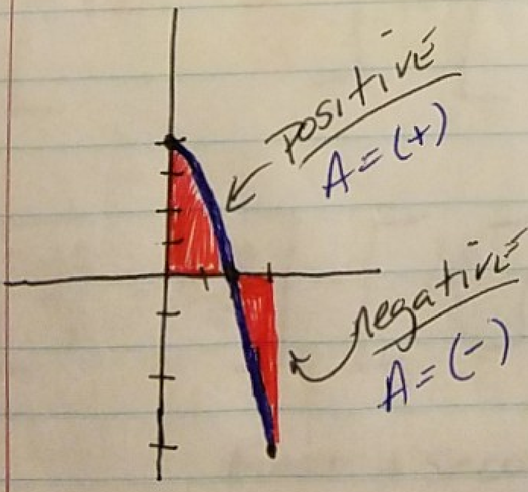
$$\frac{81}{4} + \frac{12}{4} \boxed{+c} - \frac{1}{4} + \frac{4}{4} \boxed{-c}$$

BRADY!? what is THIS CALLED?

SO WE DON'T WORRY ABOUT the "c" IN A DEFINITE INTEGRAL. Back to the PROBLEM.

$$\frac{81 + 12 - 1 + 4}{4} = \frac{96}{4} = 24$$

EXAMPLE: $f(x) = 4 - x^2 \ [0, 3]$ Hey This is THE 3rd TIME USING THIS Function



TWO PROBLEMS

$$\int_0^2 (4 - x^2)\,dx$$

$$\int_2^3 (4 - x^2)\,dx$$

THIS SHOULD BE FUN!

FIRST: $\int_0^2 (4-x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_0^2$

$\left[4(2) - \frac{(2)^3}{3}\right] - \left[4(0) - \frac{(0)^3}{3}\right]$

$\left[8 - \frac{8}{3}\right] - \left[0 - 0\right] = \frac{24}{3} - \frac{8}{3} = \frac{16}{3}$

Second: $\int_2^3 (4-x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_2^3$

$\left[4(3) - \frac{(3)^3}{3}\right] - \left[4(2) - \frac{(2)^3}{3}\right]$

$\left[12 - \frac{27}{3}\right] - \left[\frac{16}{3}\right]$

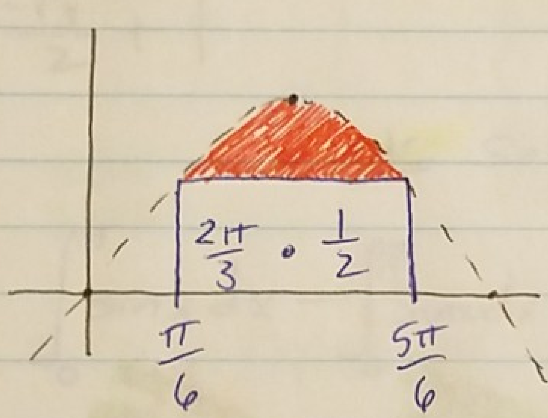
Already know this

$\left[\frac{36}{3} - \frac{27}{3}\right] - \left[\frac{16}{3}\right] = \left[\frac{9}{3} - \frac{16}{3}\right] = \left[-\frac{7}{3}\right]$

FIRST + SECOND $= \left[\frac{16}{3}\right] + \left[-\frac{7}{3}\right] = \frac{23}{3}$

EXAMPLE: $y = \sin x$ $[0, \pi]$

HEY I REMEMBER THIS ONE TOO. BUT NOW LET'S FIND AREA OF SHADED REGION.



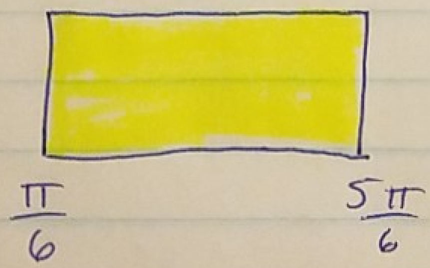
OK! $\int_0^{\pi} \sin x = \left[-\cos x\right]_0^{\pi}$

$\left[-\cos \pi\right] - \left[-\cos 0\right]$

$1 - -1 = 2$

OK NOW WE NEED TO TAKE OUT STUFF

TAKE OUT INNER BOX!



$A = l \times w$

$l = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

$w = \sin\left(\frac{\pi}{6}\right)$ or $\sin\left(\frac{5\pi}{6}\right)$

$w = \frac{1}{2}$

$A = \frac{2\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{3}$

NOW FOR OTHER PIECES! [ON THE NEXT PAGE]

$\int_0^{\frac{\pi}{6}} \sin x \, dx = \left[-\cos x\right]_0^{\frac{\pi}{6}}$ AND $\int_{\frac{5\pi}{6}}^{\pi} \sin x \, dx = \left[-\cos x\right]_{\frac{5\pi}{6}}^{\pi}$

$$\left[-\cos x\right]_0^{\frac{\pi}{6}} \qquad \left[-\cos x\right]_{\frac{5\pi}{6}}^{\pi}$$

$$\left[-\cos\left(\tfrac{\pi}{6}\right)\right]-\left[-\cos(0)\right] \qquad \left[-\cos(\pi)\right]-\left[-\cos(5\pi/6)\right]$$

$$\frac{-\sqrt{3}}{2}+1 \qquad 1+\frac{-\sqrt{3}}{2}$$

OK So. Area of SHADED REGION

$$\underbrace{\int_0^{\pi}\sin x\,dx}_{\text{Whole thing}} - \underbrace{\int_0^{\frac{\pi}{6}}\sin x\,dx}_{\text{First Piece}} - \underbrace{\int_{\frac{5\pi}{6}}^{\pi}\sin dx}_{\text{Last Piece}} - \underbrace{\left[\frac{2\pi}{3}\cdot\frac{1}{2}\right]}_{\text{Rectangle}}$$

$$2-\left[\frac{-\sqrt{3}}{2}+1\right]-\left[1+\frac{-\sqrt{3}}{2}\right]-\left[\frac{\pi}{3}\right] \approx 0.685$$

From Your Packet p. 14 & 15

Upper Limit

Integral Sign

$$\int_a^b f(x)\,dx$$

x is variable of integration

function of Integrand

Lower Limit

EX 1 EVALUATE

A.) $\int_2^5 (-3x+4)\,dx = \left[\frac{-3}{2}x^2+4x\right]_2^5$

$\left[-\frac{3}{2}(5)^2+4(5)\right] - \left[-\frac{3}{2}(2)^2+4(2)\right]$

$\left[-\frac{75}{2}+\frac{40}{2}\right] - \left[-6+8\right] = \left[\frac{-35}{2}\right]-\left[\frac{4}{2}\right]$

$= \frac{-39}{2}$

B.) $\int_{-2}^{-1} \left(u^1 - \frac{1}{u^2}\right) du = \left[\frac{1}{2}u^2 + u^{-1}\right]_{-2}^{-1}$

$\left[\frac{1}{2}(-1)^2 + \frac{1}{-1}\right] - \left[\frac{1}{2}(-2)^2 + \frac{1}{-2}\right]$

$\left[\frac{1}{2} - 1\right] - \left[2 + \frac{1}{2}\right] = -2$

C.) $\int_0^2 (t-2)\sqrt{t}\, dt$ WAIT!!

DISTRIBUTIVE PROPERTY

$\int_0^2 (t^1 - 2) t^{\frac{1}{2}}\, dt$

$\int_0^2 t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\, dt$ OK OK!! NOW PROCEED!!

$\left[\frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}}\right]_0^2 = \left[\frac{2}{5}(2)^{\frac{5}{2}} - \frac{4}{3}(2)^{\frac{3}{2}}\right] - \left[\frac{2}{5}(0)^{\frac{5}{2}} - \frac{4}{3}(0)^{\frac{3}{2}}\right]$

$= \frac{2}{5}(2)^{\frac{5}{2}} - \frac{4}{3}(2)^{\frac{3}{2}} \approx -1.508$

D.) $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\,dx = \int_{-8}^{-1} \left(\frac{1}{2}x^{\frac{2}{3}} - \frac{1}{2}x^{\frac{5}{3}}\right)dx$

$$= \left[\frac{3}{10}x^{\frac{5}{3}} - \frac{3}{16}x^{\frac{8}{3}}\right]_{-8}^{-1}$$

$$= \left[\frac{3}{10}(-1)^{\frac{5}{3}} - \frac{3}{16}(-1)^{\frac{8}{3}}\right] - \left[\frac{3}{10}(-8)^{\frac{5}{3}} - \frac{3}{16}(-8)^{\frac{8}{3}}\right]$$

CALCULATOR!?
Where Are You!?

$$\left[-\frac{3}{10} - \frac{3}{16}\right] - \left[-\frac{96}{10} - \frac{768}{16}\right] = -\frac{3}{10} \underbrace{- \frac{3}{16} + \frac{96}{10} + \frac{768}{16}}_{\text{Common Denominator.}}$$

mmm. DISTRIBUTE

$$= \frac{4569}{80}$$

E. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2 - \csc^2 x)\,dx$ ← What the Function!?

$$= \Big[2x + \cot x\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \left[2\left(\tfrac{\pi}{2}\right) + \cot\left(\tfrac{\pi}{2}\right)\right] - \left[2\left(\tfrac{\pi}{4}\right) + \cot\left(\tfrac{\pi}{4}\right)\right]$$

$$= [\pi + 0] - \left[\tfrac{\pi}{2} + 1\right]$$

$$= \tfrac{\pi}{2} - 1$$

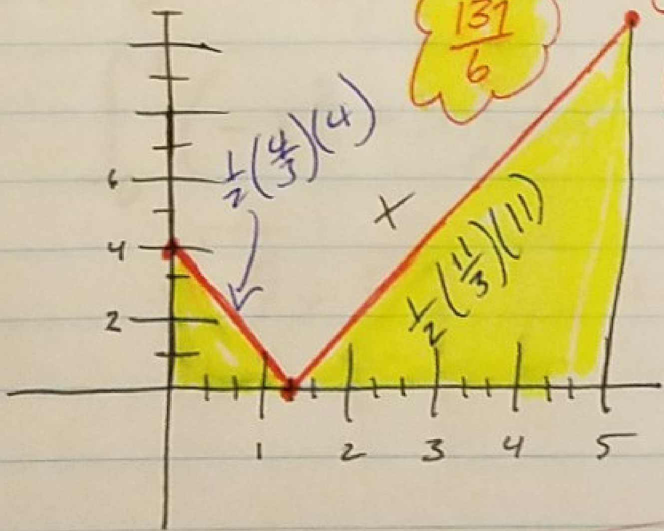
F. $\displaystyle\int_0^5 |3x-4|\,dx$

Holy Pre Calc

$|3x-4| = \left|3\left(x - \tfrac{4}{3}\right)\right|$

So $V\left(\tfrac{4}{3}, 0\right)$

$\frac{131}{6}$



2 △'s So.. EASY

BUT

$$\int_0^{\frac{4}{3}} (-3x+4)\,dx + \int_{\frac{4}{3}}^{5} (3x-4)\,dx$$

$$\left[\frac{-3x^2}{2} + 4x\right]_0^{\frac{4}{3}} + \left[\frac{3x^2}{2} - 4x\right]_{\frac{4}{3}}^5$$

$$\left[-\frac{3}{2}\left(\frac{4}{3}\right)^2 + 4\left(\frac{4}{3}\right)\right] + \left[\frac{3}{2}(5)^2 - 4(5)\right] - \left[\frac{3}{2}\left(\frac{4}{3}\right)^2 - 4\left(\frac{4}{3}\right)\right]$$

$$\left[-\frac{48}{18} + \frac{16}{3}\right] + \left[\frac{75}{2} - 20\right] - \left[\frac{48}{18} - \frac{16}{3}\right]$$

$$\frac{8}{3} + \left[+\frac{35}{2} + \frac{8}{3}\right] = \frac{137}{6}$$

SAME!

EX: $y = -x^2 + 3x$ & $y = 0$

$$\int_0^3 (-x^2 + 3x)\,dx = \left[-\frac{1}{3}x^3 + \frac{3}{2}x^2\right]_0^3$$

$$= \left[-9 + \frac{27}{2}\right] - [0 + 0] = \frac{9}{2}$$