9/21/16

IMPLICIT DIFFERENTIATION

When you CANNOT solve a function explicitly meaning you are unable to solve for y as a function of x. This when you implicitly differentiate.

Implicit Differentiation Process

1. TAKE Derivative of Both sides [w/ respect to x]
2. Collect ∀ terms with dy/dx
3. FACTOR OUT $\frac{dy}{dx}$
4. Solve for $\frac{dy}{dx}$

EX: $x^2 - xy + y^2 = 7$

1. $2x - \left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) + 2y\frac{dy}{dx} = 0$

2. $2x - \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$

$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$

$2x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

3. $2x - y + \frac{dy}{dx}(-x + 2y) = 0$

④ $2x - y + \frac{dy}{dx}(-x+2y) = 0$

$\frac{dy}{dx}(-x+2y) = y - 2x$

$$\frac{dy}{dx} = \frac{y-2x}{-x+2y}$$

EX: $2y = x^2 + \sin y$

① $2(1)y' = 2x(1) + \cos y\,(y')$

② $2y' = 2x + \cos y\,(y')$

$2y' - \cos y\,(y') = 2x$

③ $y'(2 - \cos y) = 2x$

④ $y' = \frac{2x}{2-\cos y}$ when $\cos y \neq 2$

which is Everywhere

$\cos\theta = [-1, 1]$

EX: $x^2+y^2=25$ FIND slope @ $(3,-4)$ ↑ pt

① $2x\left(\frac{dx}{dx}\right)+2y\left(\frac{dy}{dx}\right)=0$

$2x+2y\left(\frac{dy}{dx}\right)=0$

$\frac{2y\left(\frac{dy}{dx}\right)}{2y}=\frac{-2x}{2y}$

$\frac{dy}{dx}=\frac{-x}{y} \Rightarrow \frac{dy}{dx}=\frac{-3}{-4}$

$=\frac{3}{4}$ slope

eq of tangent

$y-y_1=m(x-x_1)$

$y+4=\frac{3}{4}(x-3)$

eq of normal

$y+4=-\frac{4}{3}(x-3)$

Ex! $2x^3 - 3y^2 = 8$ Find y''

① $6x^2(1) - 6y\left(\frac{dy}{dx}\right) = 0$

$$\frac{6x^2}{6y} = \frac{6y\left(\frac{dy}{dx}\right)}{6y}$$

$$\frac{x^2}{y} = \frac{dy}{dx}$$ First Derivative

$$\frac{d^2y}{dx^2} = \frac{y(2x) - x^2\left(\frac{dy}{dx}\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y(2x)}{y^2} - \frac{x^2\left(\frac{dy}{dx}\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{2x}{y} - \frac{x^2}{y^2}\left(\frac{x^2}{y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{2x}{y} - \frac{x^4}{y^3}$$ where $y \neq 0$.