Integration of Area

Riemann Sums

Back to Yesterday

18.) $f'(x) = 4x \qquad f(0) = 6$

$f(x) = \frac{4}{2}x^2 + C$

$f(x) = 2x^2 + C$

$f(0) = 6$

$6 = 2x^2 + C$

$6 = 2(0)^2 + C$

$6 = C$

$f(x) = 2x^2 + 6$

19.) $f''(x) = 2, \quad f'(2) = 5, \quad f(2) = 10$

$f'(x) = 2x + C$

$f'(2) = 5$

$5 = 2(2) + C$

$1 = C$

$f'(x) = 2x + 1$

$f(x) = x^2 + x + C$

$f(2) = 10$

$10 = (2)^2 + 2 + C$

$4 = C$

$f(x) = x^2 + x + 4$

Now It's Your Turn

$f''(x) = -2 \qquad f'(5) = 10 \qquad f(3) = -3$

$f'(x) = -2x + C$

$10 = -2(5) + C$

$20 = C$

$f'(x) = -2x + 20$

$f(x) = -x^2 + 20x + C$

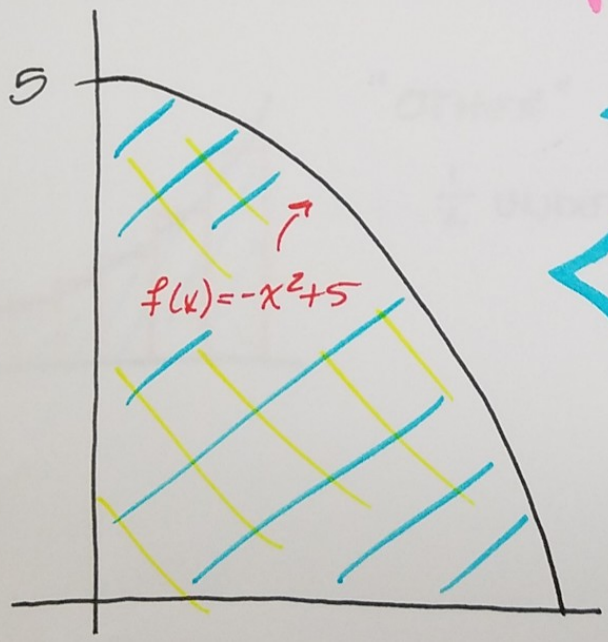
$-3 = -(3)^2 + 20(3) + C$

$-3 = -9 + 60 + C$

$C = -54$

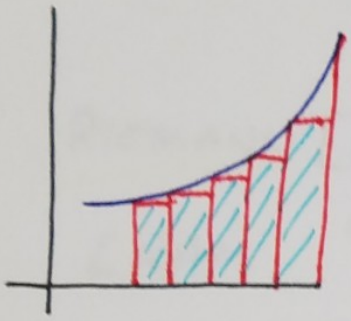
$f(x) = -x^2 + 20x - 54$

Approximating the Area Under a Curve



We are going approximate the area under a curve over a closed interval $[a, b]$.

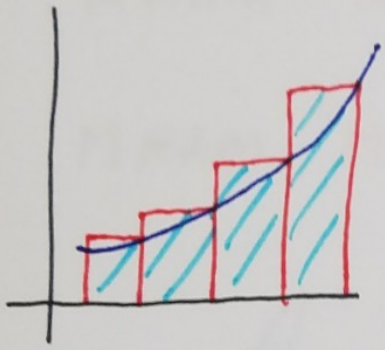
Approximation Methods



INSCRIBED RECTANGLES

BELOW THE CURVE

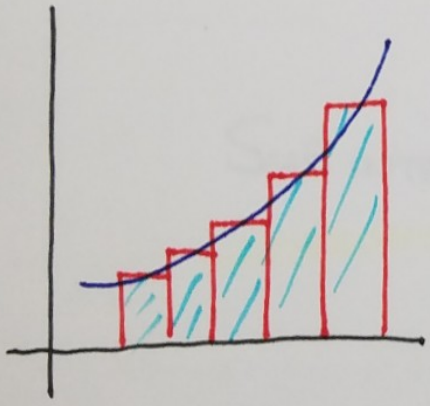
UNDERESTIMATION



CIRCUMSCRIBED RECTANGLES

OVERESTIMATION

ABOVE THE CURVE



"OTHER" ONE

$\frac{1}{2}$ UNDER AND $\frac{1}{2}$ OVER

THESE DIFFERENT APPROXIMATIONS ARE HOW WE FIND RIEMANN SUMS

RIEMANN [Ree-Mon] SUMS

LRAM "L" for LEFT ENDpoints

RRAM "R" for Right ENDpoints

MRAM "M" For MIDpoints

We ARE FINDING A NUMBER OF Areas OF RECTANGLES

$A_{rectangle} = b \cdot h$

b = WIDTH of EACH Rectangle

h = height $[f(x)]$

SUB INTERVALS - # of PIECES THAT your graph IS Broken into.

Remember $y = x^2$ $[0, 2]$ #5 on packet.

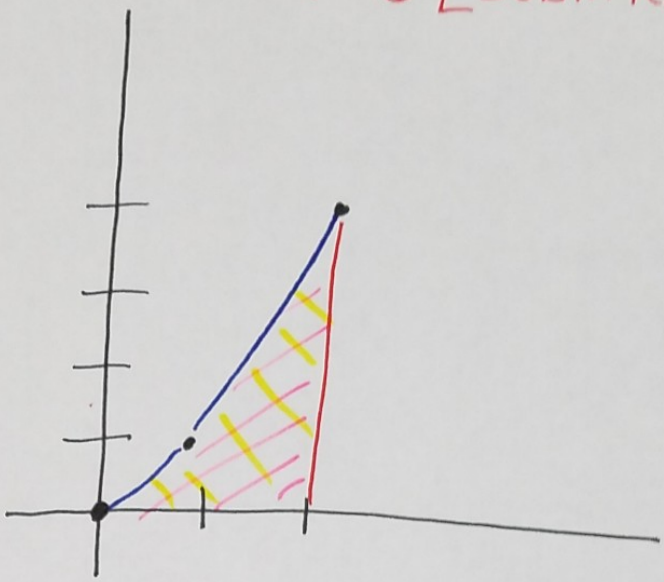
$n = 6$ [subinterval]

Width of Rectangle

$w = \frac{b-a}{n}$

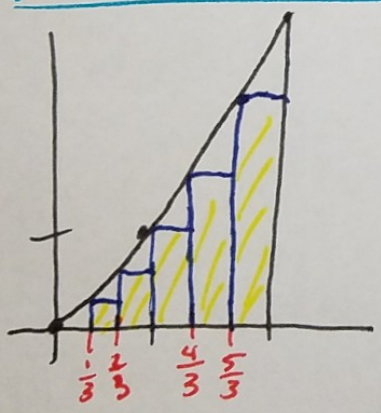
Height

$h = f(x)$



LRAM

Width $= \frac{2-0}{6} = \frac{1}{3}$



$LRAM = l \cdot w + l \cdot w + \cdots l \cdot w$

$LRAM = w[l + l + \cdots + l]$

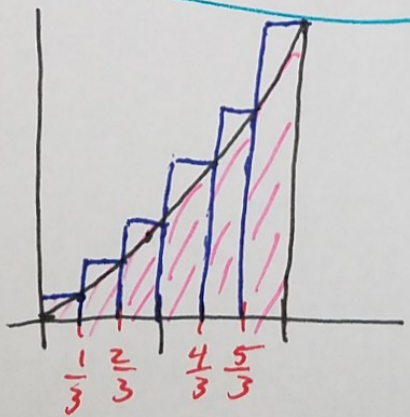
$LRAM = \frac{1}{3}\left[0 + \frac{1}{9} + \frac{4}{9} + 1 + \frac{16}{9} + \frac{25}{9}\right]$

$= \frac{1}{3}\left[\frac{55}{9}\right]$

$= \frac{55}{27}$

RRAM

Width $= \frac{2-0}{6} = \frac{1}{3}$



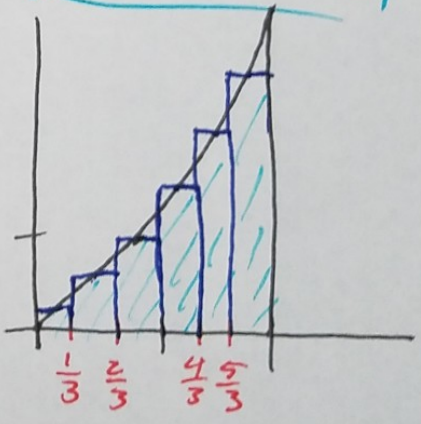
$RRAM = \frac{1}{3}\left[\frac{1}{9} + \frac{4}{9} + \frac{9}{9} + \frac{16}{9} + \frac{25}{9} + \frac{36}{9}\right]$

$RRAM = \frac{1}{3}\left[\frac{91}{9}\right]$

$RRAM = \frac{91}{27}$

MRAM

Width $= \frac{2-0}{6} = \frac{1}{3}$



$MRAM = \frac{1}{3}\left[\frac{1}{36} + \frac{1}{4} + \frac{25}{36} + \frac{49}{36} + \frac{9}{4}\right]$

$mRAm = \frac{1}{3}\left[\frac{1}{36} + \frac{9}{36} + \frac{25}{36} + \frac{49}{36} + \frac{81}{36}\right]$

$MRAM = \frac{284}{108} = \frac{143}{54}$ $+\frac{121}{36}$