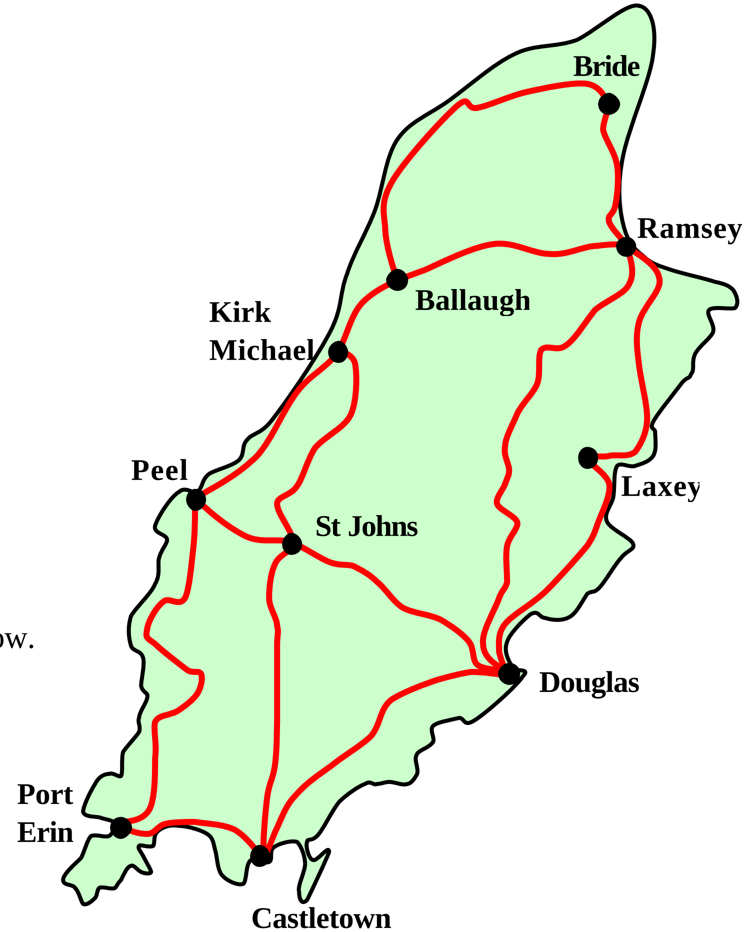
Cable TV

Data Sheet



The map shows some of the towns and roads on the Isle of Man.

A cable TV company wants to lay cables to connect the towns, laying the cable along the roads shown on the map.

They want to connect all of these towns to their cable network using the **minimum** total length of cable.

The length of the roads joining adjacent towns is given in miles in the chart below. Note that a dash (-) means there is no direct route between the towns.

Ballaugh									
9.3	**Bride**								
-	-	**Castletown**							
-	-	10.2	**Douglas**						
2.8	-	-	-	**Kirk Michael**					
-	-	-	7.7	-	**Laxey**				
-	-	-	-	6.8	-	**Peel**			
-	-	4.7	-	-	-	13.9	**Port Erin**		
6.5	4.6	-	15.4	-	9.3	-	-	**Ramsey**	
-	-	9.3	8.2	7.4	-	2.7	-	-	**St. Johns**

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Cable TV

Worksheet

The type of problem involved in the cable TV situation is often called a **minimum connector problem**. Kruskal's algorithm is one method of solving such problems.

Kruskal's Algorithm for finding a minimum connector (i.e. minimum spanning tree)

Step 1 List the edges in order of increasing weights.

Step 2 Start with the edge with the smallest weight.

Step 3 From the remaining edges, choose the one with the smallest weight which **does not form a cycle.** (If there are 2 shortest edges, choose either.)

Step 4 Repeat Step 3 until all the vertices are connected.

Carry this out for the cable TV problem. First list the edges in order of length below.

Then follow steps 2 to 4, using the map below to show your solution.

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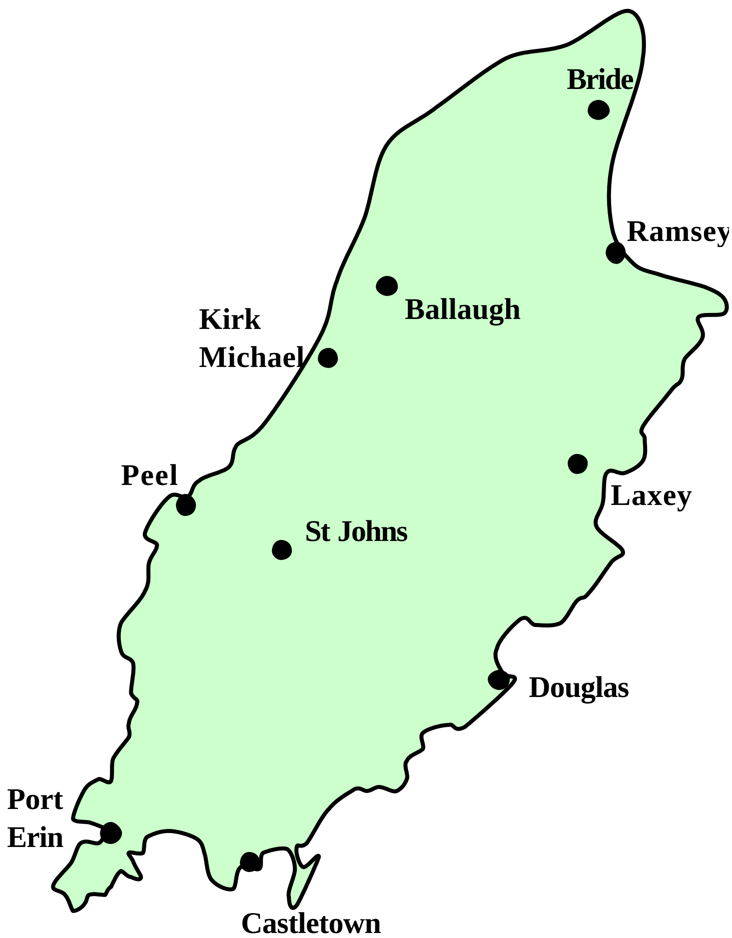

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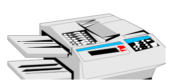


Prim's algorithm gives an alternative method for minimum connector problems.

Prim's Algorithm for finding a minimum connector (i.e. minimum spanning tree)

Step 1 Starting from any vertex, join it to the nearest adjacent vertex.

Step 2 Join the next nearest vertex to those already included, provided that this **does not form a cycle.** (If there are 2 nearest vertices, choose either.)

Step 3 Repeat Step 2 until all the vertices are included.

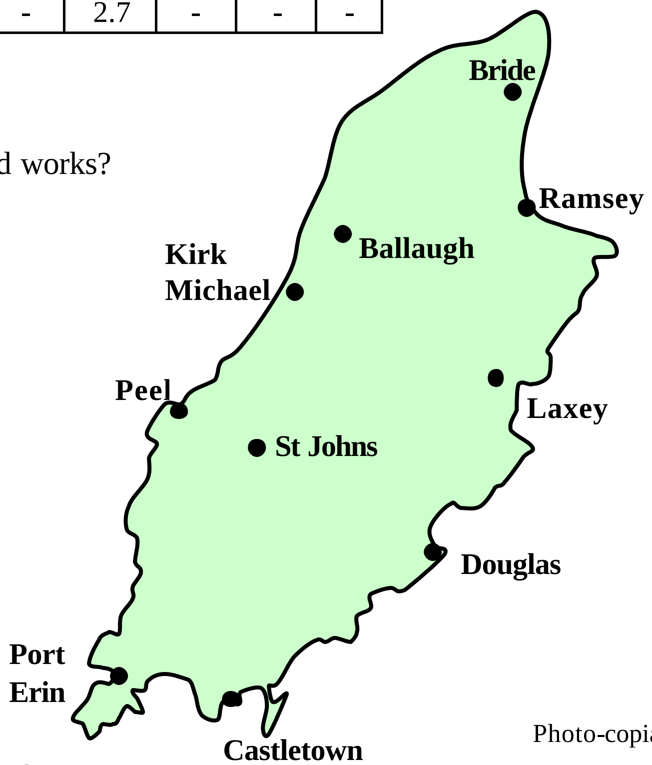
This method can be carried out using a network diagram and matrix/table (see below).
To use the table, choose a starting vertex, say B_1 and label it as 1 above the table.
Delete row B_1 and look for the smallest entry in column B_1. This is the 2.8 in row K.
This means B_1K is in the solution. Label K as 2 (in table) and draw B_1K (on map).
Now delete row K and look for the smallest entry in column B_1 or K. This is 6.5 in column B_1 and row R. Label R as 3 and draw B_1R on the map.
Continue in this way until all the vertices are connected.
You should find the solution is identical to that found using Kruskal's algorithm.

	B_1	**B_2**	**C**	**D**	**K**	**L**	**P_1**	**P_2**	**R**	**S**
B_1	-	9.3	-	-	2.8	-	-	-	6.5	-
B_2	9.3	-	-	-	-	-	-	-	4.6	-
C	-	-	-	10.2	-	-	-	4.7	-	9.3
D	-	-	10.2	-	-	7.7	-	-	15.4	8.2
K	2.8	-	-	-	-	-	6.8	-	-	7.4
L	-	-	-	7.7	-	-	-	-	9.3	-
P_1	-	-	-	-	6.8	-	-	13.9	-	2.7
P_2	-	-	4.7	-	-	-	13.9	-	-	-
R	6.5	4.6	-	15.4	-	9.3	-	-	-	-
S	-	-	9.3	8.2	7.4	-	2.7	-	-	-

B_1 = Ballaugh
B_2 = Bride
C = Castletown
D = Douglas
K = Kirk Michael
L = Laxey
P_1 = Peel
P_2 = Port Erin
R = Ramsey
S = St Johns

Can you explain how the matrix method works?

Can you think of the advantages or disadvantages that the solution may have in practice?



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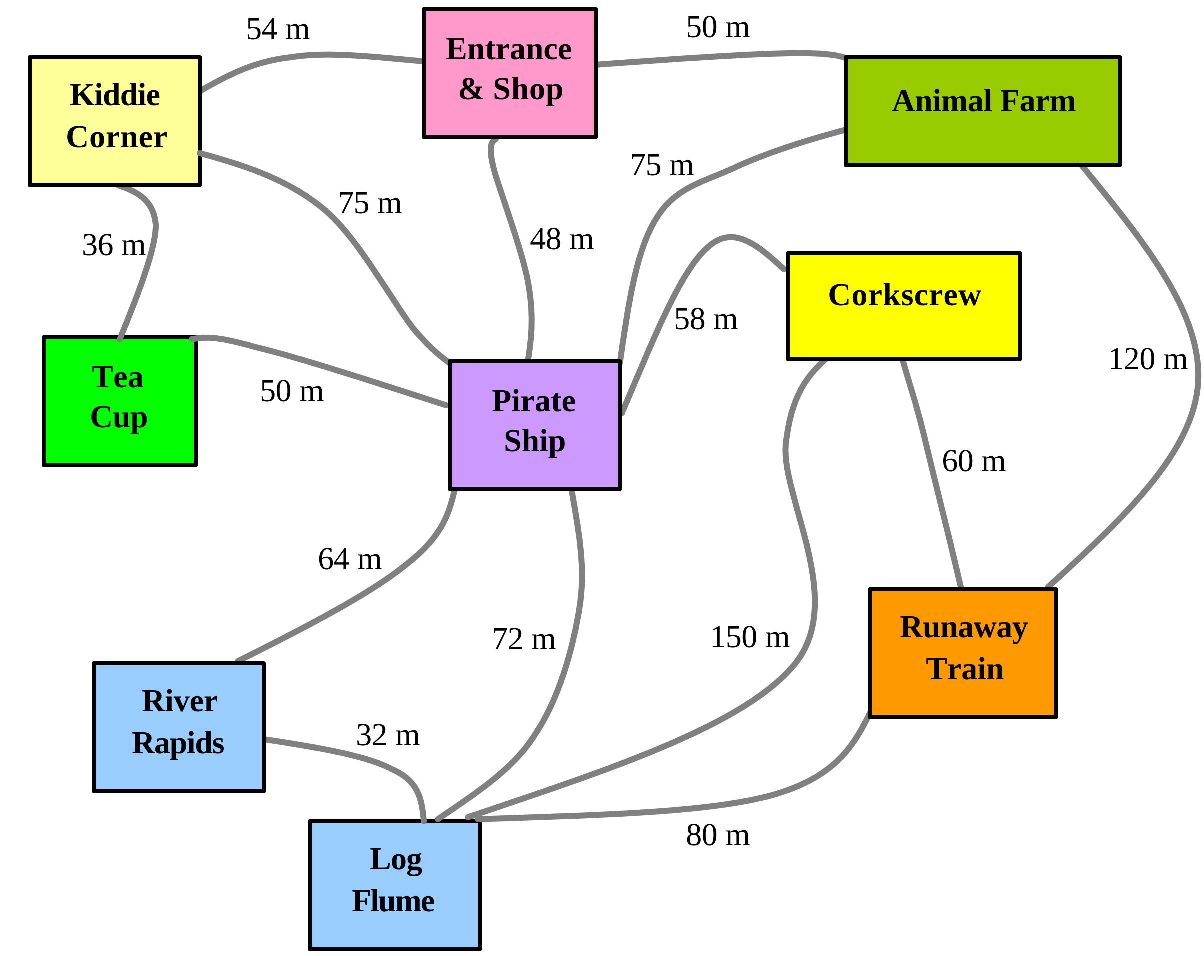
Theme Park Paths

Worksheet

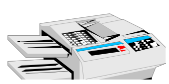
The plan below shows the lengths of paths linking parts of a theme park.



The manager of the theme park wants to widen and re-surface some of these paths to provide better access to the rides for wheelchairs. His aim is to provide better paths to connect all parts of the theme park, but using the minimum total length possible.

Find the minimum spanning tree for this network.

Write a paragraph explaining the advantages and disadvantages that your solution may have in the real situation.



Teacher Notes

Unit Advanced Level, Using and applying decision mathematics

Notes on Activity

This activity can be used to introduce Kruskal's and Prim's algorithms for finding minimum connectors or used at the end of the course as a revision exercise. The accompanying Powerpoint presentation includes the same examples and it is suggested that this and the Data and Worksheets are used as follows:

- First use the Data Sheet and first slide of the Powerpoint presentation to introduce the context. Explain that the cable TV company intends to run its cables alongside the existing roads (shown in red on the map) and that the table at the bottom of the Data Sheet gives the distances in miles.

- Ask students (individually or in pairs) to find one way of connecting all the towns on the map, trying to keep the total length of the connecting edges as low as possible. They can use the map on the Data Sheet to do this, transferring the lengths they need from the mileage chart.

- Discuss the results (noting the minimum length of cable used) and the need for a systematic way of tackling such problems when they involve complex networks.

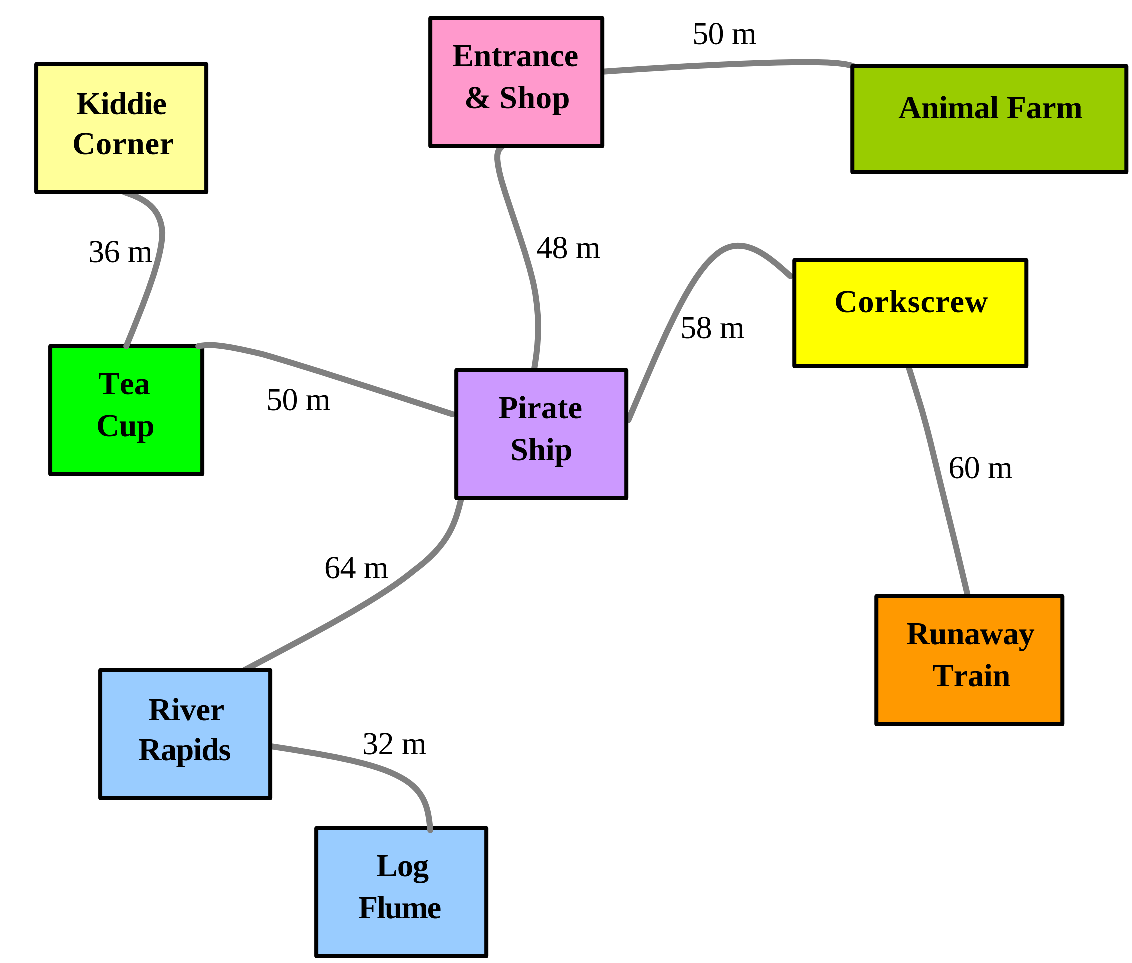
- Use the second Powerpoint slide to outline Kruskal's algorithm. If necessary introduce or remind students of related vocabulary such as network, vertex, edge, weight, cycle, spanning tree, minimum connector etc. The third and fourth slides show the solution to the cable TV problem – you could use this to demonstrate the method or to check students' solutions after they have tried this themselves using the worksheet on page 2. The solution shows that the minimum possible total length of cable is 53.3 miles.

- Discuss possible advantages and disadvantages that the solution may have in the real context. For example, a possible disadvantage is that the routes chosen may include some difficult terrain.

- Prim's algorithm is outlined on the fifth slide of the Powerpoint presentation. Students can be asked to do this using the copy of the Isle of Man map and/or the adjacency matrix on page 3. Slides 6 to 8 on the Powerpoint presentation give the matrix solution and the resulting network – again this can be used for demonstrating the method or checking students' solutions. Discuss the advantage of using a matrix – this means that Prim's algorithm can be carried out using a computer.

- Compare and discuss the two algorithms before asking students to use one or both of them to solve the Theme Park Paths problem.

Answer to Theme Park Paths problem



	1	3	5	6	4	2	8	7	9
	E	**A**	**K**	**C**	**T**	**P**	**RR**	**RT**	**L**
E	-	50	54	-	-	48	-	-	-
A	50	-	-	-	-	75	-	120	-
K	54	-	-	-	36	75		-	-
C	-	-	-	-	-	58	-	60	150
T	-	-	36	-	-	50	-	-	-
P	48	75	75	58	50	-	64	-	72
RR	-	-	-	-	-	64	-	-	32
RT	-	120	-	60	-	-	-	-	80
L	-	-	-	150	-	72	32	80	-

E = Entrance & Shop
A = Animal Farm
K = Kiddie Corner
C = Corkscrew
T = Tea Cup
P = Pirate Ship
RR = River Rapids
RT = Runaway Train
L = Log Flume

The total length of pathway in the minimum connector
= 48 + 50 + 50 + 36 + 58 + 60 + 64 + 32 = **398 metres**