

FAIR DIVISION: LESSON 1

Introduction
The Adjusted Winner Procedure

FAIR DIVISION INTRODUCTION

- ◉ No one wants to be treated unfairly
- ◉ With the 1.2 million divorces every year in the US alone, crises such as that in the Middle East, it is certainly worth wondering how mathematics can ensure fair and equitable resolutions of such conflicts.



FAIR DIVISION INTRODUCTION

- Disagreement is both a good thing and a bad thing. It typically is the reason why there is a conflict, but allows us to see the importance of the issue to each party
- Fair division procedures have been being developed for the past 60 years.

FAIR DIVISION INTRODUCTION

- ◉ We will discuss several methods of fair division including adjusted winner, divide and choose, and cake cutting procedures.
- ◉ Some will yield allocations that are proportional: each player receives what he or she believes is his or her fair share
- ◉ Others will be envy-free: each player perceives his or her piece to be at least tied for the largest.

ADJUSTED WINNER PROCEDURE

- ◉ A fair division procedure introduced by Steven Brams and Alan Taylor in 1993. It works only for two players.
- ◉ We are now going to look at the 1998 merger between two pharmaceutical giants, Glaxo Wellcome and SmithKline Beecham.

GlaxoWellcome



ADJUSTED WINNER PROCEDURE

◎ Steps:

1. Each party distributes 100 points over the items in a way that reflects their relative worth to the party
2. Each item is initially given to the party that assigned it more points. Each party then adds their points. The party with the fewest points is now given each item on which both parties placed the same number of points (ie ties).

ADJUSTED WINNER PROCEDURE

3. Typically when the points are added, they are not equal. So something must be transferred from the player with more points to the player with fewer points. This may be a whole item or a fractional part of an item.
4. We decide which item to transfer based on smallest **point ratio**. We do the point ratios for all the times that the player with points has. We do the point ratio by the player with more points for the item/player with less points for the item.

ADJUSTED WINNER PROCEDURE

5. Equation to equalize points in case a fractional part of an item must be transferred.
6. Complete answer in a SENTENCE!

ADJUSTED WINNER PROCEDURE

- ◎ Here are 5 social issues the two companies considered paramount:
 1. The name the combined company would use
 2. The location of the headquarters
 3. The question of who would serve as chairman
 4. The question of who would serve as CEO
 5. The question of where the necessary layoffs would come from

ADJUSTED WINNER PROCEDURE

- ◉ The starting point can be difficult when dealing with issues as opposed to objects.
- ◉ So, the parties involved quantify the importance it attaches to getting its way on each of the issues.

ADJUSTED WINNER PROCEDURE

◉ Here is the breakdown of the 100 points:

Point Allocations		
Issue	Glaxo	SmithKline
Name	5	10
Headquarters	25	10
Chairmen	35	20
CEO	15	35
Layoffs	20	25
Total	100	100

ADJUSTED WINNER PROCEDURE

- ◉ Glaxo Wellcome initially wins headquarters (25) and chairmen (35) because it has “given” these issues more points.
- ◉ SmithKline Beecham is initially given name (10), CEO (35), and layoffs (25).

ADJUSTED WINNER PROCEDURE

- ◉ SmithKline Beecham has $10+35+25=70$ of its points
- ◉ Glaxo Wellcome has only $25+35=60$
- ◉ Now we will have to transfer issues from SmithKline to Glaxo

ADJUSTED WINNER PROCEDURE

- ⊙ Layoffs have a point ratio of $25/20=1.25$
- ⊙ Name has a point ratio of $10/5=2.00$
- ⊙ CEO has a point ratio of $35/15=2.33$
- ⊙ Layoffs has the lowest point ratio, so we will start to transfer that item first.

ADJUSTED WINNER PROCEDURE

- ⊙ If we transferred the entire lay off (worth 25 points to SmithKline and 20 to Glaxo) would give Glaxo more points ($60+20=80$) ($70-25=45$)
- ⊙ So, the entire layoff cannot be transferred
- ⊙ Algebra is going to help us equalize the points

ADJUSTED WINNER PROCEDURE

- We will think of SmithKline Beecham as retaining some fraction x of the issue and Glaxo Wellcome receiving the complementary fraction of $1-x$ of the same issue.

ADJUSTED WINNER PROCEDURE

- ◉ Because x is the fraction of the issue that SmithKline Beecham retains, the number of points they get from the issue is x times 25.
- ◉ That would leave Glaxo Wellcome with $1-x$ times 20.

ADJUSTED WINNER PROCEDURE

- If we want a fraction that will make SmithKline Beecham's total points and Glaxo Wellcome's total points, the x must satisfy the following equation

$$10 + 35 + 25x = 25 + 35 + 20(1 - x)$$

ADJUSTED WINNER PROCEDURE

- ⊙ Solve the equation
- ⊙ $x = 7/9$
- ⊙ Inserting $7/9$ back into the equation, we see that

$$60 + 20(2/9) = 45 + 25(7/9)$$

- ⊙ Or approximately 64 points for each side.



ADJUSTED WINNER PROCEDURE

- ◎ **Theorem: Properties of the Adjusted Winner Allocation**
 - Equitable: both players receive the same number of points
 - Envy-free: neither player would be happier with what the other received.
 - Pareto-optimal: no other allocation, arrived at by any means, can make one party better off without making the other party worse off.

ADJUSTED WINNER EXAMPLE 2

- ◉ The following point allocation is based on interviews with experts and analysis of documentation on the parties' positions.

Item	Palestinian Points	Israeli Points
Temple Mount	48	22
Western Wall	4	31
Muslim/Christian Quarters	22	9.5
Israeli neighborhoods	1	19.5
Jewish/Armenian Quarters	6	18
Palestinian Neighborhoods	19	0

ADJUSTED WINNER EXAMPLE 2

- ◎ **Palestinians get:**

- Muslim and Christian quarters, Palestinian neighborhoods, and 71% of *Al-Haram al-Sharif*

- ◎ **Israelis get:**

- Jewish and Armenian quarters, Western Wall, Israeli neighborhoods, and 29% of *Al-Haram al-Sharif*

YOUR TURN!

- ⦿ You will work in your groups to create a scenario in which adjusted winner can be used to come to a fair settlement.
- ⦿ You must divide at least 5 objects.
- ⦿ Award points so they do not end up equal (I want you to have an equation to solve).
- ⦿ Have your problem on one sheet and your answer worked out on another, we will be passing problems around for practice.
- ⦿ What I deem to be the “best” problem will show up on your quiz which is Tuesday!