Mean Value Theorem
Average Value of a Function
2ND Fundamental Theorem

MEAN VALUE THEOREM – f is continuous on $[a, b]$ then $\exists$ c in $[a,b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

Average Value of a Function – f is integratable on $[a,b]$, then.

$$\text{Average Value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

2^{ND} Fundamental Theorem of Calculus

2^{ND} F.T.C - if f is continuous open interval I containing "a", then $\forall x$ in the interval

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

OK, let's go slowly.

$$F(x) = \int_{\frac{\pi}{2}}^{x} \cos t\,dt$$ $\Rightarrow$ FIND THE DERIVATIVE

We are looking for "essentially speaking" $\cos t\,dt$ BUT in terms of "x" AND NOT "t".

$$\boxed{F'(x) = \frac{dF}{dx}}$$

$$F(x) = \int_{\frac{\pi}{2}}^{x} \cos t \, dt \Rightarrow \sin t \Big|_{\frac{\pi}{2}}^{x}$$

$$F(x) = \sin x - \sin\left(\frac{\pi}{2}\right)$$

$$F(x) = \sin x - 1$$

$$F'(x) = \cos x \,(dx)$$

What do you notice?

ok, level 2.

$$F(x) = \int_{\frac{\pi}{2}}^{x^3} \cos t \, dt$$

THIS is Different. THIS HAS AN x^3... CHAIN RULE

$$F(x) = \sin t \Big|_{\frac{\pi}{2}}^{x^3}$$

$$F(x) = \sin x^3 - \sin\left(\frac{\pi}{2}\right)$$

$$F(x) = \sin x^3 - 1$$

$$F'(x) = \cos(x^3)(3x^2)$$

CHAIN RULE

So LET'S EXPLAIN what we ARE Doing.

Substitute **t** with the upper bound and multiply by the derivative of the upper bound. (dt)

McStone

$$\int_x^3 \tan t \, dt$$

$$\int_b^a f(x)dx = -\int_a^b f(x)dx$$

$$= -\int_3^x \tan(t)\,dt =$$

$$-\left[\sec^2(x) - \sec^2(3)\right]$$

CONSTANT

$F(x) = -\sec^2 x + \sec^2(3)$

$F'(x) = -\tan x$

$$F(x) = \int_0^x \sqrt{t^2+1}\, dt$$

$$F'(x) = \sqrt{x^2+1}$$

plug in the "x"

$$F(x) = \int_1^x \frac{20}{v^2}\, dv$$

alyssa

$$F'(x) = \frac{20}{x^2}$$

$$F(x) = \int_2^x (t^3+2t-2)\, dt$$

sam

$$F'(x) = (x^3+2x-2)$$

$$F(x) = \int_0^{x^4} \sin t\, dt$$

Jane

$$F'(x) = \sin x^4 (4x^3)$$