Calculus AB Unit #3 Notes
Applications of Derivatives: Important Theorems

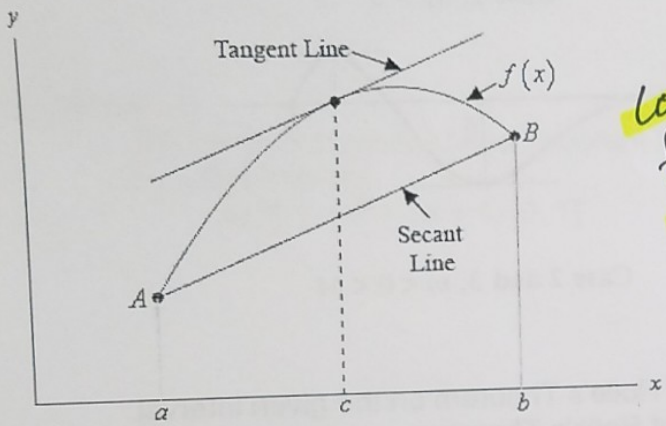
Mean Value Theorem

If the function f is continuous on the closed interval [a, b] and differentiable on the open interval (a, b), then there exists at least one number c in the open interval (a, b) such that:

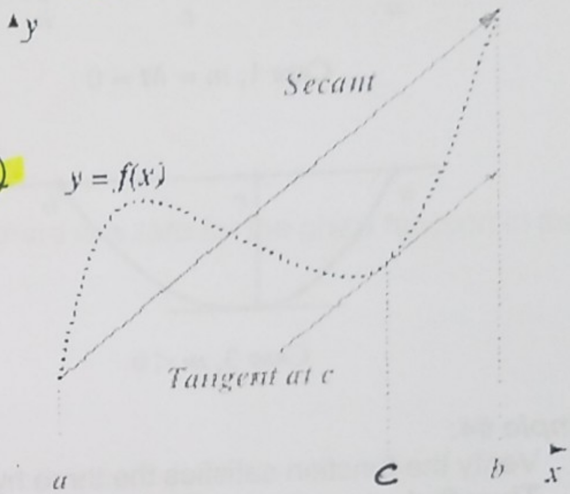
$(x, f(x))$

$(a, f(a))\ (b, f(b))$

$$\underbrace{f'(c)}_{\text{slope of tangent}} = \underbrace{\frac{f(b) - f(a)}{b - a}}_{\text{slope of secant}}$$



Looking for equal slopes



$f(x) = -3x^{-1} + 4$

Example #1:

If the function f is defined on [1, 3] by f(x) = 4 – 3/x, show that the MVT can be applied to f and find a number c which satisfies the conclusion.

$f'(x) = -3(-1)x^{-2} + 0$ ① f is cont [1, 3] AND Diff. (1, 3)

$f'(x) = \frac{3}{x^2}$

Example #2:

Sketch a graph of the function f if $f(x) = \begin{cases} x - 2, & for\ x \leq 1 \\ x^2, & for\ x > 1 \end{cases}$.

Show that f fails to satisfy the MVT on the interval [-2, 2].

f is not continuous @ x = 1

f is not Diffentiable @ x = 1

Example #3:

Suppose that $s(t) = t^2 - t + 4$ is the position of the motion of a particle moving along a line.

$s'(t) = 2t - 1$

a) Explain why the function s satisfies the hypothesis of the MVT.

S(t) is Continuous

S(t) is Differentiable

b) Find the value of t in [0, 3] where instantaneous velocity is equal to the average velocity.

AROC SLOPE

$t = \frac{3}{2}$

10/4/16 Notes Graphing Sketching

Day 1

Ex: 1

$f(x) = -3x^{-1} + 4$

$f(1) = -3(1)^{-1} + 4$

$f(1) = -3 + 4$

$f(1) = 1$

$f(3) = -3(3)^{-1} + 4$

$f(3) = -1 + 4$

$f(3) = 3$

$(1,1)$ & $(3,3)$

$f'(x) = \dfrac{3}{x^2}$

MVT $\exists c = x$

$\dfrac{3}{x^2} = \dfrac{3-1}{3-1}$

$\dfrac{3}{x^2} = 1 \Rightarrow 3 = 1 \cdot c^2$

$3 = c^2$

$\sqrt{3} = c$

why not $-\sqrt{3}$?

NO!

REMEMBER f is Defined $[1, 3]$

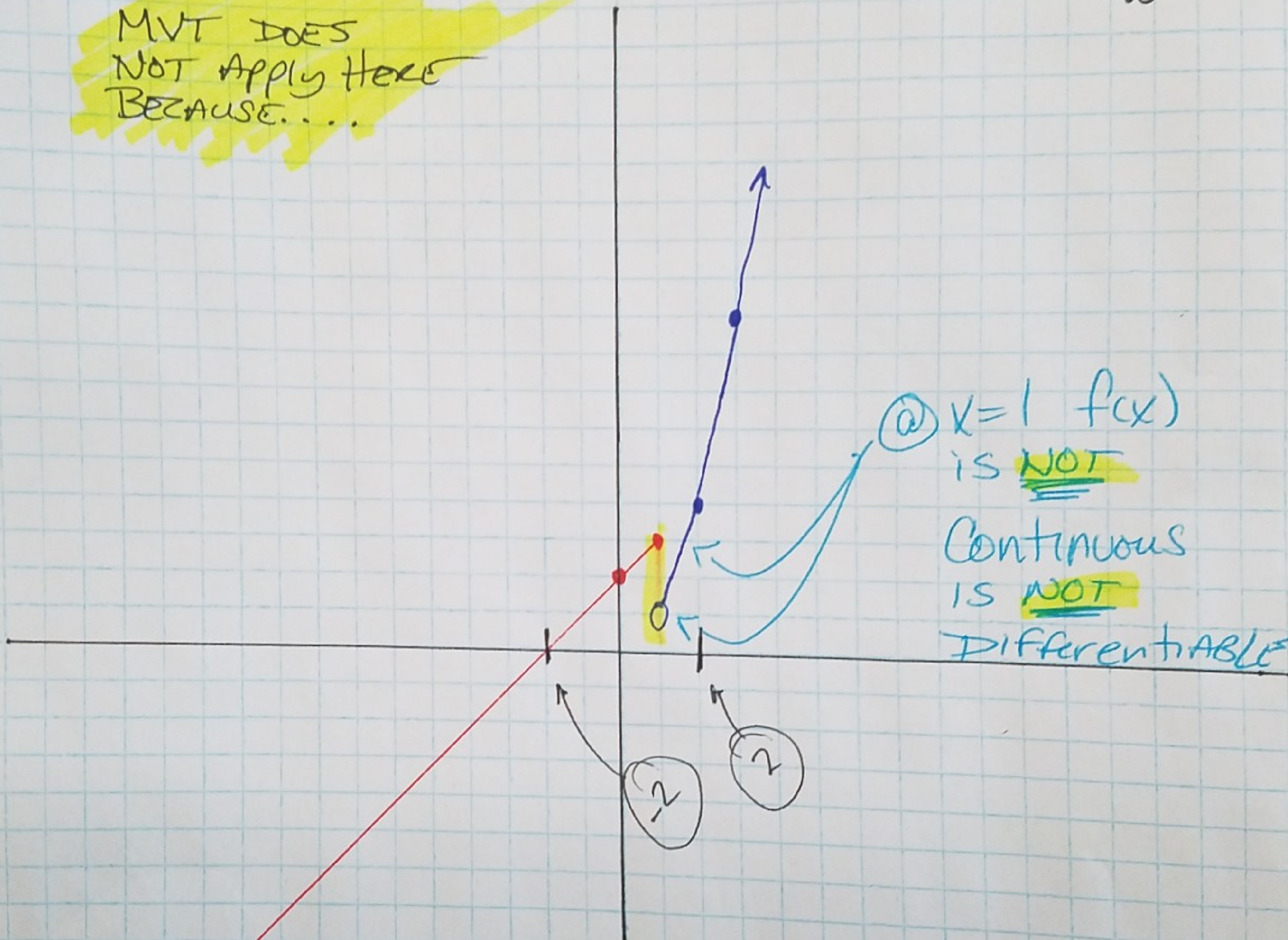
AROC

SLOP

EX: 2

$$f(x) = \begin{cases} x+2 & \text{iff } x \le 1 \\ x^2 & \text{iff } x > 1 \end{cases}$$

MVT DOES NOT APPLY HERE BECAUSE....



Ex: 3 $\quad S(t) = t^2 - t + 4$

$S'(t) = 2t - 1$

$S(0) = 0^2 - 0 + 4$

$S(0) = +4$

$(0, 4)$

$S(3) = 3^2 - 3 + 4$

$S(3) = 9 - 3 + 4$

$S(3) = 10$

$(3, 10)$

<u>MVT</u>

$$2t - 1 = \frac{10-4}{3-0} = \frac{6}{3} = 2$$

$2t - 1 = 2$

$2t = 3$

$t = \frac{3}{2}$

EXAmple 4 ~~Rolle's Theorem~~

$f(x) = x^2 - 4x + 1 \quad [0,4]$

$f'(x) = 2x - 4$

f is continuous $[0,4]$ Yes!

f is Differentiable $(0,4)$ Yep!

$f(0) = 0^2 - 4(0) + 1$

$f(0) = 1$

$f(4) = 4^2 - 4(4) + 1$

$f(4) = 1$

$f(0) = f(4)$

So...

$2x - 4 = 0$

$2x = 4$

$x = 2$

Must $\exists c$ in $(a,b) \ni f'(c) = 0$

AND THERE it is

ARC 5/3

EXAMPLE #5

① $f(x) = \sin(2\pi x)$ $[-1, 1]$

② $f'(x) = \cos(2\pi x)(2\pi)$

f is continuous @ $[-1, 1]$ Yes!

f is Differentiable $[-1, 1]$ Yes!

$f(-1) = \sin(-2\pi)$ $\quad$ $f(1) = \sin(2\pi)$

$f(-1) = 0$ $\quad$ $f(1) = 0$

$f(-1) = f(1)$ Yes! So....

$f'(x) = \cos(2\pi x)(2\pi)$

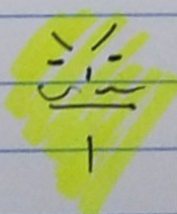
$0 = 2\pi \cos(2\pi x)$

$0 = \cos(2\pi x)$

$\cos^{-1}(0) = 2\pi x$

where is $\cos^{-1}(0)$

$$\underbrace{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{-\pi}{2}, \frac{-3\pi}{2}}_{\text{ugh four problems....}}$$



$\frac{3\pi}{2} = 2\pi x$

$\boxed{\frac{3}{4} = x}$ $\quad$ $\boxed{x = \frac{-3}{4}}$

$\frac{\pi}{2} = 2\pi x$

$\boxed{\frac{1}{4} = x}$ $\quad$ $\boxed{x = -\frac{1}{4}}$