Optimization Day 1

Applied Min/Max

FIND TWO (2) NUMBERS WHOSE SUM IS 20 AND WHOSE PRODUCT IS AS LARGE AS POSSIBLE.

Number 1 – UNKNOWN [x]

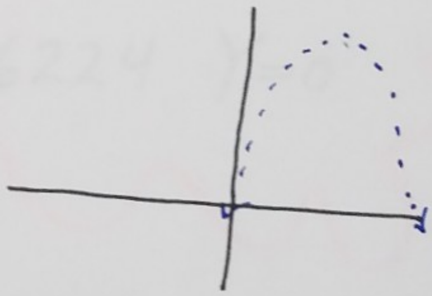
Number 2 – $x + y = 20$

$-x \quad -x$

$\boxed{y = 20 - x}$

$f(x) = x(20-x)$

$f(x) = 20x - x^2$



$f'(x) = 20 - 2x$

$0 = 20 - 2x$

$2x = 20$

$\boxed{x = 10}$

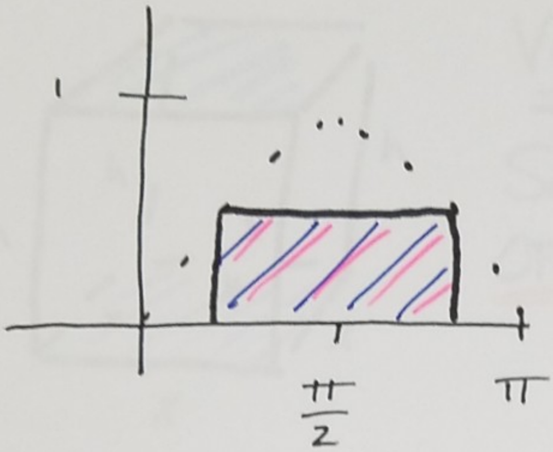
$x = 10$

$y = 20 - x$

$y = 20 - 10$

$y = 10$

EXAMPLE: LARGEST AREA OF RECTANGLE.



$\sin x \; [0, \pi]$

Need

$\left.\begin{array}{l}\text{Length}\\ \text{Height}\end{array}\right\}$ of Rectangle

$A = l \times w$

$l = \pi - 2x$

$w = \sin x$

$A = (\pi - 2x)\sin x$

$A' = -2\sin x + (\pi - 2x)\cos x$

$0 = -2\sin x + (\pi - 2x)\cos x$

$X = 0.71046224 \quad Y = 0$

$l = \pi - 2(x)$

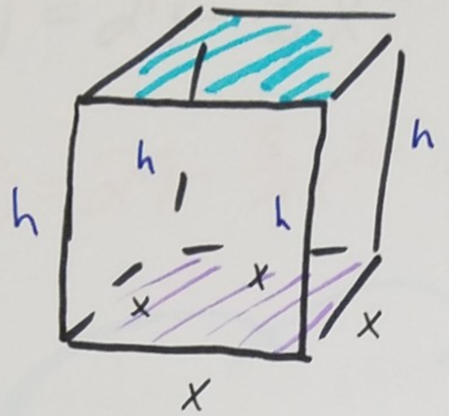
$l = 1.721$

$w = \sin(x)$

$w = 0.652$

SOMETIMES WE USE CALculators.... JUST FOR FUN.

EXAmple: MAximum Volume



$V = l \times w \times h$

$SA = 108\,in^2$

OPEN TOP

Square BASE Box

We gonna Fill it!

$SA = 108\,in^2 = (A_{BASE}) + 4(A_{SIDE})$

$\text{Volume} = l \times w \times h$

$l = w$ [Square BASE]

$V = x^2 h$

$SA = 108\,in^2 = x^2 + 4(xh)$

$$\frac{108\,in^2 - x^2}{4x} = h$$

$$V = x^2\left(\frac{108 - x^2}{4x}\right)$$

$$V = \frac{108x^2 - x^4}{4x}$$

$$V = 27x - \frac{1}{4}x^3$$

$V = 27x + -\frac{1}{4}x^3$

$V' = 27 - \frac{3}{4}x^2$

$0 = 27 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 27$

$x^2 = 36$

$x = \pm 6$

$\cancel{x = -6}$

Why?

$h = \frac{108 - x^2}{4x} = \frac{108 - 6^2}{4(6)} = \frac{72}{24} = 3$

$V = x^2 h = (6)^2(3) = \boxed{108\,in^3}$

smallest.

Example: Looking for largest sum.

$\boxed{\text{Two Numbers whose Product is 192}}$

Number 1 → UNKNOWN → x

Number 2 → $x \cdot \square = 192$

$\square = \frac{192}{x}$

$Sum = x + \left(\frac{192}{x}\right)3$

[minimum]

$S = x + \left(\frac{192}{x}\right)3$

$S = x + \frac{576}{x}$

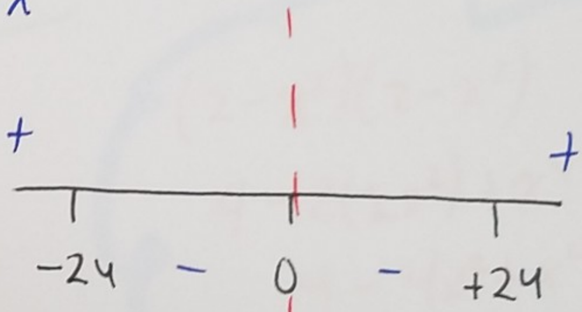
$S' = 1 + \frac{-576}{x^2}$

$S' = \frac{x^2 - 576}{x^2}$

$S' = \frac{(x+24)(x-24)}{x^2}$

Zero @ $x = \pm 24$

UNDEFINED @ $x = 0$

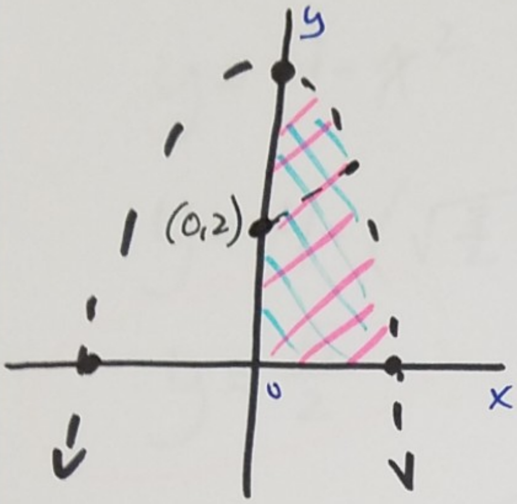


$x = 24$

$\square = \frac{192}{24}$

$\square = 8$

Example: $y = 4 - x^2$ $(0,2) \leftarrow$ closest to point.



MINIMUM DISTANCE

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

pt $(0,2)$ pt (x,y)

$$d = \sqrt{(x-0)^2 + (y-2)^2}$$

$$d = \sqrt{(x-0)^2 + (4 - x^2 - 2)^2}$$

$$d = \sqrt{(x-0)^2 + (2 - x^2)^2}$$

$$(2 - x^2)(2 - x^2)$$

$$4 - 2(2x^2) + x^4$$

$$4 - 4x^2 + x^4$$

Perfect Square Trinomial

$$d = \sqrt{x^2 + 4 - 4x^2 + x^4}$$

$$d = \sqrt{x^4 - 3x^2 + 4}$$

$$d = \underbrace{(x^4 - 3x^2 + 4)}^{\frac{1}{2}}$$

Derivative of inside only set = to zero

$$4x^3 - 6x = 0$$

$$2x(2x^2 - 3) = 0$$

$$x = 0 \quad x = \pm\sqrt{\tfrac{3}{2}}$$

Sign CHART

$$\begin{array}{ccccccc} - & \text{MIN} & + & \text{MAX} & - & \text{MIN} & + \\ & -\sqrt{\tfrac{3}{2}} & & 0 & & \sqrt{\tfrac{3}{2}} & \end{array}$$

So. . .

$y = 4 - x^2$

$y = 4 - \left(\sqrt{\frac{3}{2}}\right)^2$

$y = \frac{5}{2}$

$y = 4 - x^2$

$y = 4 - \left(-\sqrt{\frac{3}{2}}\right)^2$

$y = \frac{5}{2}$

$\left(\sqrt{\frac{3}{2}}, \frac{5}{2}\right) \left(-\sqrt{\frac{3}{2}}, \frac{5}{2}\right)$