Quiz Conversation

- Definite Integral
- F.T.C. (Number 1)
- MVT (Integrals)
- Average Value
- F.T.C (Number 2)
- Trapezoid Rule

Integral Rule's

$$\int a\,dx = ax + C$$

$$\int x\,dx = \frac{1}{2}x^2 + C$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int \frac{1}{x}dx = \ln x + C$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\int a^x dx = \left(\frac{1}{\ln a}\right)a^x + C$$

First Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

NOTE:

$f(x) \Rightarrow F(x)$

function — Integrated function

$$\int_0^1 (x+4)\,dx$$

$$\frac{1}{2}x^2 + 4x \Big|_0^1$$

$$F(b) = \frac{1}{2}(1)^2 + 4(1) = \frac{9}{2}$$

$$F(a) = \frac{1}{2}(0)^2 + 4(0) = 0$$

$$\frac{9}{2} - 0 = \frac{9}{2}$$

Mean Value Theorem

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

Ex $\int_0^2 2x\,dx = 2\int_0^2 x\,dx$

$$2\cdot\left(\frac{1}{2}\right)x^2\Big|_0^2$$

$$X^2\Big|_0^2$$

$F(b) = 2^2 = 4$

$F(a) = 0^2 = 0$

$F(b) - F(a) = 4$

$4 = f(c)(2-0)$

$4 = f(c)(2)$

$2 = f(c)$

$2 = 2x$

$1 = x$

$$\int_{-x}^{x} f(x)\,dx = \int_{-x}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx$$

$$= -\int_{0}^{-x} f(x)\,dx + \int_{0}^{x} f(x)\,dx$$

Average Value THeorem

$$\frac{1}{b-a}\int_{a}^{b} f(x)\,dx = f(c)$$

$$\frac{1}{2-0}\int_{0}^{2} 2x\,dx = f(c)$$ ← Previous Problem

$$\frac{4}{2} = f(c)$$

$$2 = f(c)$$

2ND Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_0^x f(t)\,dt$$

$$\frac{d}{dx}\int_0^{x^3} \sec^2 t\,(dt)$$

$$3x^2\sec^2(x^3)$$

$$\int_{-x}^{x} \cos t\,dt = \int_{-x}^{0} \cos t\,dt + \int_0^x \cos t\,dt$$

$$= -\int_0^{-x} \cos t\,dt + \int_0^x \cos t\,dt$$

$$= -\cos(-x)(-1) + \cos x\,(1)$$

$$\cos(-x) + \cos(x)$$

Trapezoid Rule

$$\int f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+\cdots 2(f_{x_{n-1}}) + f(x_n)\right]$$

b - upper

a - lower

n = # SUBINTERVALS

$$\int_0^{\pi} \sin x\,dx$$

$n = 4$

$$\frac{\pi-0}{2(4)} = \frac{\pi}{8}$$

$$\frac{\pi}{8}\left[\sin(0)+2\sin\left(\frac{\pi}{2}\right)+2\sin(\pi)+2\sin\left(\frac{3\pi}{2}\right)+\sin(\pi)\right]$$

$$= \frac{\pi}{8}\left[0+2\cdot\frac{1}{\sqrt{2}}+2+2\cdot\frac{1}{\sqrt{2}}+0\right]$$

$$= \frac{\pi}{8}\left[2+\frac{4}{\sqrt{2}}\right]$$

$$\int_0^{\sqrt{\frac{\pi}{2}}} \cos(x^2)\,dx$$

$$\sqrt{\frac{\pi}{128}}\left(\cos\left[(0)^2\right] + 2\cos\left[\left(\sqrt{\frac{\pi}{32}}\right)^2\right] + 2\left[\cos\left[\left(\frac{2\sqrt{\pi}}{\sqrt{32}}\right)^2\right] + 2\cos\left[\left(3\sqrt{\frac{\pi}{32}}\right)^2\right] + \cos\left[\left(4\sqrt{\frac{\pi}{32}}\right)^2\right]\right.\right)$$

$$\sqrt{\frac{\pi}{128}}\left(1 + 2\cos\left(\frac{\pi}{32}\right) + 2\cos\left(\frac{4\pi}{32}\right) + 2\cos\left(\frac{9\pi}{32}\right) + 2\cos\left(\frac{16\pi}{32}\right)\right)$$

$$\sqrt{\frac{\pi}{128}}\left(6.10691\,5087\right) = 0.9567353767$$

TRAPEZOID RULE

UNEQUAL SUBINTERVALS

SLIGHTLY Different

x	1	4	6	7
f(x)	10	20	80	30

WITH TABLE

$\frac{b-a}{2n}$ where $n=1$

$$\frac{4-1}{2(1)}(10+20)+\frac{6-4}{2(1)}(20+80)+\frac{7-6}{2(1)}(80+30)$$

$$\frac{3}{2}(30)+1(100)+\frac{1}{2}(110)$$

$$45+100+55$$

$$200$$