From Yesterday

$f(x) = \sin x$ FIND $f'(x) = ?$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

SAME [GCF]

$$\lim_{h\to 0} \frac{\sin x\cos h + \cos x \sin h - \sin x}{h}$$

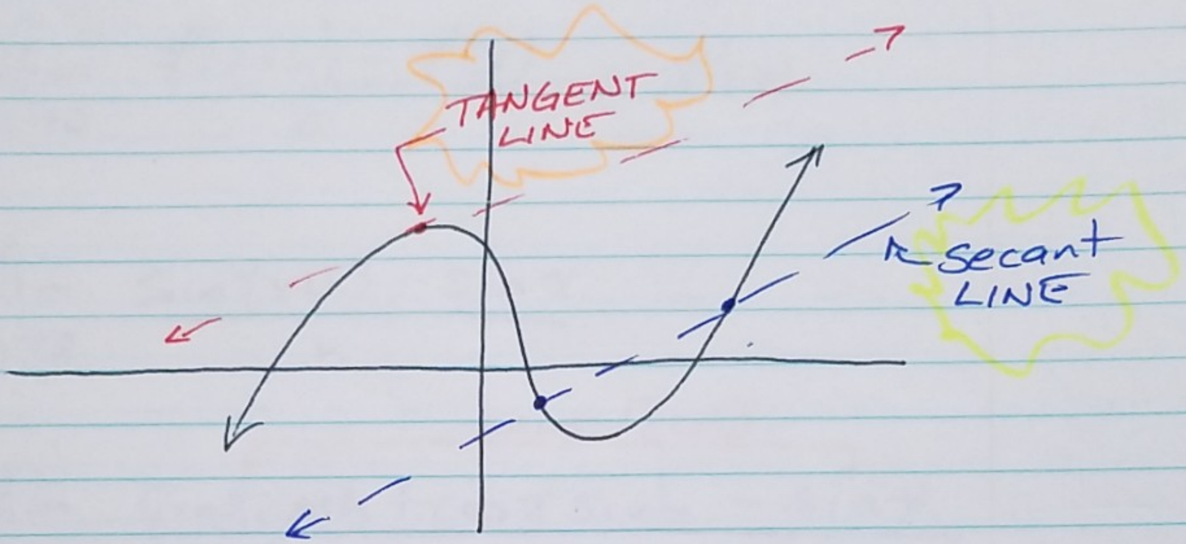
$$\lim_{h\to 0} \frac{\cos x \sin h + \sin x(\cos h - 1)}{h}$$

$$\lim_{h\to 0} \left(\frac{\cos x}{1}\right)\underbrace{\left(\frac{\sin h}{h}\right)}_{1} + \left(\frac{\sin x}{1}\right)\underbrace{\left(\frac{\cos h - 1}{h}\right)}_{0}$$

$= \cos x$

$f'(x) = \cos x$

Rates of Change



AVERAGE RATE OF CHANGE → SLOPE [SECANT LINES]
A.R.O.C.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

SLOPE OF CURVE'S TANGENT AT A POINT

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

INSTANTANEOUS RATE of CHANGE

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

NORMAL LINE TO A CURVE → LINE PERPENDICULAR TO TANGENT LINE @ a GIVEN POINT.

EX: $f(x) = 4 - x^2$

A) FIND A.R.O.C. $[1, 3]$

$f(1) = 4 - 1^2 = \boxed{3}$

$f(3) = 4 - 3^2 = \boxed{-5}$

$m = \frac{-5-3}{3-1} = \frac{-8}{2} = -4$

B) FIND SLOPE AND TANGENT LINE at $X = 2$

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$f(a) = 4 - a^2$

$f(a+h) = 4 - (a+h)^2$

B) Continued

$$m = \lim_{h \to 0} \frac{[4-(a+h)^2] - [4-a^2]}{h}$$

REMEMBER THAT HERE $\boxed{x = 2 = a}$

$$\lim_{h \to 0} \frac{4 - (a^2 + 2ah + h^2) - 4 + a^2}{h}$$

$$\lim_{h \to 0} \frac{\cancel{4} \cancel{-a^2} - 2ah - h^2 \cancel{-4} \cancel{+a^2}}{h}$$

$$\lim_{h \to 0} \frac{-2a\cancel{h}}{\cancel{h}} - \frac{h^{\cancel{2}}}{\cancel{h}} = -2a = -4$$

So Tangent Line is

Point Slope form

$$y - y_1 = m(x - x_1)$$

So pretty

$m = -4$
pt $\to (2, 0)$

$$y = -4(x-2)$$

(C) FIND THE NORMAL CURVE.

SLOPE OF TANGENT = -4

SO SLOPE OF NORMAL IS ITS opposite RECIPROCAL

OPPOSITE RECIPROCAL !?

→ OPPOSITE SIGN

→ Flip the FRACTION

SLOPE OF NORMAL

$$\boxed{m = \frac{1}{4}}$$

NORMAL TO A CURVE at $x=2$

$$y - 0 = \frac{1}{4}(x-2)$$

$$y = \frac{1}{4}(x-2)$$

(D) FIND I.R.O.C.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = -2x$$

$$\boxed{f'(x) = -2x}$$