Example 1

A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the disturbed water changing?

REMEMBER

$$A' = \frac{dA}{dt}$$

KNOW

$A_{circle} = \pi r^2$

radius $\Rightarrow r = 4\,ft$

Radius increasing [Rate]

$\frac{dr}{dt} = 1\,\frac{ft}{s}$

$A = \pi R^2$

$A' = 2\pi R\left(\frac{dr}{dt}\right)$

$A' = 2\pi(4)\left(1\,\frac{ft}{s}\right)$

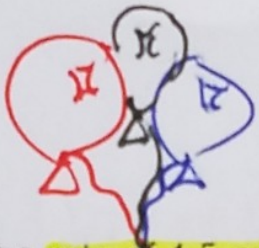
$A' = 8\pi\,\frac{ft^2}{s}$

[[~~TOTAL AREA~~ of H_2O increases at a Rate of $8\pi\,\frac{ft^2}{sec}$]]

NEED TO KNOW

$2\,\frac{dA}{dt} = ?$

Example 2



Air is being pumped into a spherical balloon at a rate of 4.5 cubic inches per minute. Find the rate of change of the radius when the radius is 2 inches.

FIND $\frac{dr}{dt}$ @ $r = 2$ in

"pumped into a spherical balloon"

Volume!

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4.5 \frac{in^3}{min}$

$V' = \left(\frac{4}{3}\right)(3)\pi r^2 \left(\frac{dr}{dt}\right)$

$4.5 \frac{in^3}{min} = \left(\frac{4}{3}\right)(3)\pi (2)^2 \left(\frac{dr}{dt}\right)$

$4.5 \frac{in^3}{min} = 16\pi \left(\frac{dr}{dt}\right)$

$\frac{dr}{dt} = \frac{9}{2}\left(\frac{1}{16\pi}\right)\frac{in}{min}$

$\frac{dr}{dt} = \frac{9}{32\pi}\frac{in}{min}$

Example 3

Water runs into a conical tank at the rate of 2 ft^3/min. The tank stands with the point down and has a height of 10 feet and a base diameter of 10 feet. How fast is the water level rising when the water is 6 feet deep?

Rate → $\frac{dV}{dt} = 2\frac{ft^3}{min}$

Find $\frac{dh}{dt}$ when $h = 6$ ft.

$V = \frac{1}{3}\pi r^2 h$

$\frac{\text{Radius}}{\text{Height}} = \frac{5}{10}$

$10r = 5h$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ ← height

$V = \frac{1}{3}\pi\left(\frac{h^2}{4}\right)\frac{h}{1}$

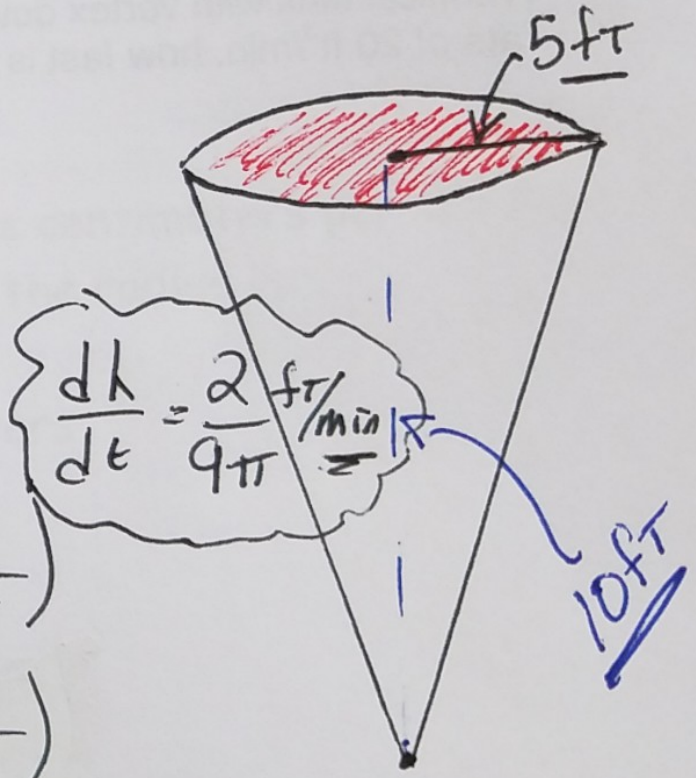
$V = \frac{1}{3}\left(\frac{1}{4}\right)\pi(h^3)$

$V = \frac{1}{12}\pi(h^3)$

$V' = \frac{\pi}{12}(3)h^2\left(\frac{dh}{dt}\right)$

$2\frac{ft^3}{min} = \frac{\pi}{12}(3)(6)^2\left(\frac{dh}{dt}\right)$

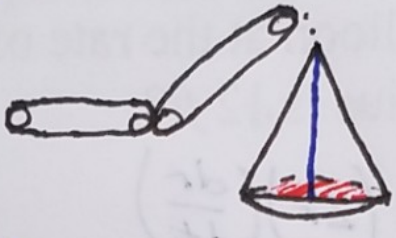
$\frac{dh}{dt} = \frac{2}{9\pi}\frac{ft}{min} = \frac{2}{9\pi}$



Example 4

Example 4

Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always 3/8 of the base of the diameter. How fast are the height and radius changing when the pile is 4 m high?



$\frac{dV}{dt} = 10 \frac{m^3}{min}$

FIND $\frac{dh}{dt}$ & $\frac{dr}{dt}$ @ $h = 4m$

$V = \frac{1}{3}\pi r^2 h$

$h = \frac{3}{8} d$

$d = 2r$

$h = \frac{3}{8}(2r) = \frac{3}{4} r \Rightarrow \frac{4}{3} h = r$

$V = \frac{1}{3}\pi \left(\frac{4}{3}h\right)^2 h$

$V = \frac{1}{3}\pi \left(\frac{16}{9}h^2\right) h$

$V = \frac{16\pi}{27} h^3$

$\frac{dV}{dt} = \frac{16\pi}{27}(3) h^2 \left(\frac{dh}{dt}\right)$

$10 \frac{m^3}{min} = \frac{16\pi}{27}(3)(4)^2 \left(\frac{dh}{dt}\right)$

$10 \frac{m^3}{min} = \frac{768\pi}{27} \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{45}{128\pi} \frac{m}{min}$

$\frac{3}{4}(r) \rightarrow \frac{3}{4} \cdot \frac{dr}{dt}$

$\frac{3}{4} \frac{dr}{dt} = \frac{dh}{dt}$

$\frac{dr}{dt} = \frac{4}{3}\left(\frac{dh}{dt}\right)$

$\frac{dr}{dt} = \frac{4}{3}\left(\frac{45}{128\pi}\right)$

$\frac{dr}{dt} = \frac{15}{32\pi} \frac{m}{min}$

Example 5

The radius of a right circular cylinder is increasing at a rate of 2 *in/min* and the height is decreasing at a rate of 3 *in/min*. At what rate is the volume changing when the radius is 8 *in* and the height is 12 *in*? Is the volume increasing or decreasing?

$\frac{dh}{dt} = -3\frac{in}{min}$

$\frac{dr}{dt} = 2\frac{in}{min}$

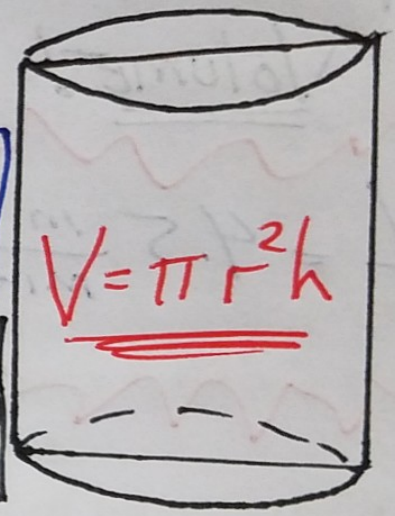
FIND $\frac{dV}{dt}$ $r = 8in$ $h = 12in$

$V = \pi r^2 h$

$V' = \pi\left[r^2\left(\frac{dh}{dt}\right) + h(2r)\frac{dr}{dt}\right]$

$\frac{dV}{dt} = \pi\left[8^2\left(-3\frac{in}{min}\right) + 12(2)(8)(2)\right]$

$\frac{dV}{dt} = 192\pi\ in^3/min$ POSSITIVE WHICH MEANS . . . INCREASING



Example 6