

Sequences.

TWO TYPES EXPLICIT

Arithmetic $\Rightarrow a_n = a_1 + d(n-1)$

Geometric $\Rightarrow a_n = a_1(r)^{n-1}$

Domain \Rightarrow Set of all positive integers.

↳ Not always all positive integers to ∞ ; However all the numbers are positive.

RECURSIVE [NOW-NEXT]

$$\text{NEXT} = \text{NOW} + \text{RATE}$$

or

$$\text{NEXT} = \text{NOW} \cdot \text{RATE}$$

OTHER TIMES THEY ARE SOMETHING ELSE!

$$a_n = 3 + (-1)^n$$

$$\begin{aligned}a_1 &= 3 + (-1)^1 = 2 \\a_2 &= 3 + (-1)^2 = 4 \\a_3 &= 3 + (-1)^3 = 2 \\a_4 &= 3 + (-1)^4 = 4\end{aligned}$$

These answers
are
2, 4, 2, 4, ...

what about

$$\frac{2}{2}, \frac{4}{3}, \frac{6}{4}, \frac{8}{5}, \frac{10}{6}, \dots$$

look at the change in each number
in sequence.

$$\frac{2 \cdot 1}{1+1} \quad \frac{2 \cdot 2}{1+2} \quad \frac{2 \cdot 3}{1+3} \quad \frac{2 \cdot 4}{1+4} \quad \frac{2 \cdot 5}{1+5}$$

$$a_n = \frac{2n}{n+1}$$

OR

$$\frac{1}{1}, \frac{4}{3}, \frac{9}{7}, \frac{16}{15}$$

$$\frac{1^2}{2-1} \quad \frac{2^2}{4-1} \quad \frac{3^2}{8-1} \quad \frac{4^2}{16-1}$$

$$a_n = \frac{n^2}{2^n - 1}$$

In this Chapter we will investigate.
CONVERGENCE

CONVERGENCE when sequences have terms approaching **limiting values**.

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \text{etc.}$$

$$a_n = \left(\frac{1}{2}\right)^n$$

$$\lim_{n \rightarrow \infty} a_n = L \quad \begin{array}{l} \text{if } \forall \alpha > 0 \exists M > 0 \text{ such} \\ \text{that } |a_n - L| < \alpha \text{ whenever} \\ n > M \end{array}$$

Limit of A Sequence

$$\lim_{x \rightarrow \infty} f(x) = L \quad \begin{array}{l} \text{if } a_n \text{ is a sequence such that} \\ f(n) = a_n, \text{then} \end{array}$$

$$\lim_{n \rightarrow \infty} a_n = L$$

Ex: $a_n = \left(1 + \frac{1}{n}\right)^n$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e \quad \text{Prior Knowledge}$$

So. $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$

Ex: $a_n = 3 + (-1)^n$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} (3 + (-1)^n) \quad \text{Divergence}$$

Because this ALTERNATES Between 2 and 4
the Sequence Diverges. Diverges \Rightarrow DNE [Limit]

Ex: $a_n = \frac{n^2}{2n-1}$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left(\frac{n^2}{2n-1}\right) \quad \text{L'Hospital!}$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^2}{2n-1}\right) = \lim_{n \rightarrow \infty} \left(\frac{2n}{2}\right) \Rightarrow \infty \quad \text{Diverges.}$$

Ex: $a_n = \frac{n^2}{2^n - 1}$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n^2}{2^n - 1}$$

L'Hospital
 $\frac{\infty}{\infty} \checkmark$ indeterminate
form

$$\lim_{n \rightarrow \infty} a_n = 0$$

~~Converges~~

AP Calculus BC
Notes 2.1 Sequences

Sequence- a function whose domain is the set of positive integers. Its elements are called "terms". n is the number term and used to generate the terms. a_n is the general term or the " n th term".

n	1	2	3	4	5	n
a_n	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{n}$

Arithmetic Sequence

Ex: $-3, 1, 5, 9, 13, \dots$ $d =$

Ex: $13, 7.5, 2, -3.5, \dots$ $d =$

Geometric Sequence $a_n = -3 + (4)(n-1)$

$a_n = 13 + (-5.5)(n-1)$

Ex: $16, 8, 4, 2, 1, \dots$ $r =$

$$a_n = 16 \left(\frac{1}{2}\right)^{n-1}$$

Ex: $2, 5, \frac{25}{2}, \frac{125}{4}, \dots$ $r =$

$$a_n = 2 \left(\frac{5}{2}\right)^{n-1}$$

Limit of a Sequence

Let L be a real number. L is the limit of a sequence $\{a_n\}$, written $\lim_{n \rightarrow \infty} a_n = L$.

- If the limit EXISTS, then the sequence Converges.
- If the limit DNE, then the sequence Diverges.

Ex 1) Given the n^{th} term, find the limit, if it exists.

a) $a_n = \frac{1}{n}$ $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$

b) $a_n = \left(1 + \frac{1}{n}\right)^n$ $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$ \downarrow

* $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow \infty} \left(\frac{x+1}{x}\right)^x = e$ PreCalculus Unit on Exp Growth Decay.
 we also showed this in Unit 1.

c) $a_n = 3 + (-1)^n$

$$\lim_{n \rightarrow \infty} 3 + (-1)^n \quad 3 + (-1)^{\infty}$$

$$3 + (-1)^1 = 2$$

$$3 + (-1)^2 = 4$$

$$3 + (-1)^3 = 2$$

$$3 + (-1)^4 = 4$$

Divergent

END Behaviour
Precalc.

d) $a_n = \sin n$

$$\lim_{n \rightarrow \infty} \sin(n)$$

Range $[-1, 1]$ Divergent

$$\lim_{n \rightarrow \infty} \frac{n}{1-2n}$$

$$\text{L'Hospital} \quad \frac{1}{-2}$$

$$\frac{n}{-2n} = -\frac{1}{2}$$

$$\begin{aligned} & \div n \cdot \frac{n}{1-2n} = \frac{1}{\frac{1}{n}-2} \\ & = -\frac{1}{2} \end{aligned}$$

Ex 2) Determine convergence.

a) $a_n = \frac{1-5n^4}{n^4+8n^3} = -5 \quad \frac{-5n^4}{n^4} = \boxed{\frac{-5}{1}}$

$$\frac{-20n^3}{4n^3} \rightarrow \frac{-60n^2}{12n^2} \rightarrow \frac{-120n}{24n} \rightarrow \frac{-120}{24} = \boxed{\frac{-5}{1}}$$

b) $a_n = \frac{n^2-2n+1}{n-1} \quad \lim_{n \rightarrow \infty} \frac{(n-1)(n-1)}{(n-1)} \quad \lim_{n \rightarrow \infty} n-1 \quad \text{Divergent.}$

c) $a_n = \left(\frac{n+1}{2n}\right)\left(1-\frac{1}{n}\right) \quad \lim_{n \rightarrow \infty} \left(\frac{n+1}{2n}\right)\left(\frac{n-1}{n}\right) = \frac{n^2-1}{2n^2} = \frac{1}{2}$

d) $a_n = \frac{n^2}{2^n - 1}$

$$\frac{2n}{\ln(2)(2)^n} = \frac{2}{n\ln 2} = 0$$

e) $a_n = 1 + \frac{(-1)^n}{n}$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \left(1 + \frac{(-1)^n}{n} \right) = 1$$

Properties of Limits of Sequences

Let $\lim_{n \rightarrow \infty} a_n = L$ and $\lim_{n \rightarrow \infty} b_n = K$

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- $\lim_{n \rightarrow \infty} (a_n \pm b_n) = L \pm K$

- $\lim_{n \rightarrow \infty} ca_n = cL$ c is \mathbb{R}

- $\lim_{n \rightarrow \infty} (a_n b_n) = LK$

- $\lim_{n \rightarrow \infty} \left(\frac{a_n}{b_n} \right) = \frac{L}{K}$ $K \neq 0$

Squeeze Theorem for Sequences

If $\lim_{n \rightarrow \infty} a_n = L$ & $\lim_{n \rightarrow \infty} b_n = L$ and there exists an integer N such that $a_n \leq c_n \leq b_n$ for all $n > N$, then $\lim_{n \rightarrow \infty} c_n = L$.

$$\frac{1}{2^n} \leq \frac{1}{n} \quad n > 0$$

Ex 3) Given $c_n = \frac{1}{2^n}$, find $\lim_{n \rightarrow \infty} c_n$

$$\lim_{n \rightarrow \infty} 0 \leq \lim_{n \rightarrow \infty} \frac{1}{2^n} \leq \lim_{n \rightarrow \infty} \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} \frac{1}{2^n} = 0$$

$$0 \leq \lim_{n \rightarrow \infty} \frac{1}{2^n} \leq 0$$

Absolute Value Theorem for Sequences

Given the sequence $\{a_n\}$. If $\lim_{n \rightarrow \infty} |a_n| = 0$, $\lim_{n \rightarrow \infty} a_n = 0$.

Ex 4) $a_n = -\frac{1}{n}$

$$\lim_{n \rightarrow \infty} \left| -\frac{1}{n} \right| = 0 = \lim_{n \rightarrow \infty} \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

Ex 5) Given each sequence, find the n^{th} term.

a) $2, \frac{4}{3}, \frac{8}{5}, \frac{16}{7}, \frac{32}{9}, \dots$

$$\frac{2}{1}, \frac{2 \cdot 2}{3}, \frac{2 \cdot 2 \cdot 2}{5}, \frac{2 \cdot 2 \cdot 2 \cdot 2}{7}, \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{9}$$

b) $-2, \frac{8}{2}, -\frac{26}{6}, \frac{80}{24}, -\frac{242}{120}, \dots$

$$\frac{-2}{1}, \frac{8}{3}, \frac{-26}{15}, \frac{80}{45}, \frac{-242}{135}$$

$$\frac{(-1)^n(3^n-1)}{n!}$$

$$\boxed{\frac{2^n}{2n+1}}$$