THE TRAPEZOIDAL RULE

Let f be continuous on $[a, b]$. The Trapezoidal Rule for approximating

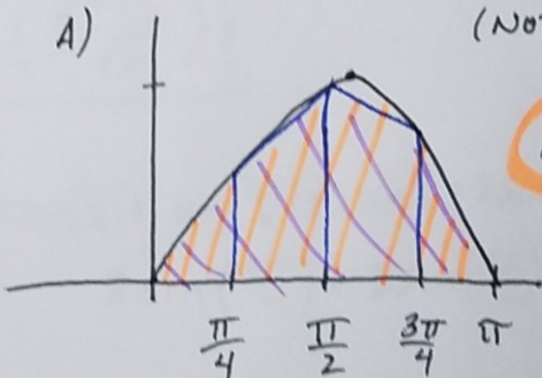
$$\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + \cdots 2f(x_{n-1}) + f(x_n)\right]$$

(Moreover, as $n \to \infty$, the right hand side $\to \int_a^b f(x)\,dx$)

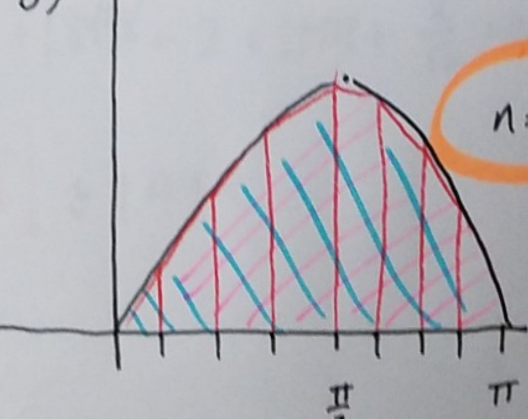
Ex: $\displaystyle\int_0^{\pi} \sin x\,dx$

A) (NOT 2 SCALE)



Compare
$n=4$
$n=8$

B)



A.) $n=4$

$$\frac{b-a}{2n} = \frac{\pi - 0}{2 \cdot 4} = \frac{\pi}{8}$$

$$\int_0^{\pi} \sin x \, dx = \frac{\pi}{8}\left[\sin(0) + 2\sin\frac{\pi}{4} + 2\sin\frac{\pi}{2} + 2\sin\frac{3\pi}{4} + \sin\pi\right]$$

$$= \frac{\pi}{8}\left[0 + 2\left(\frac{1}{\sqrt{2}}\right) + 2(1) + 2\left(\frac{1}{\sqrt{2}}\right) + 0\right]$$

$$= \frac{\pi}{8}\left[0 + \frac{2}{\sqrt{2}} + 2 + \frac{2}{\sqrt{2}} + 0\right]$$

$$= \frac{\pi}{8}\left[\frac{2}{\sqrt{2}} + \frac{2\sqrt{2}}{\sqrt{2}} + \frac{2}{\sqrt{2}}\right]$$

$$= \frac{\pi}{8}\left[\frac{4 + 2\sqrt{2}}{\sqrt{2}}\right] = \frac{\pi}{8}\left[2\left(\frac{2+\sqrt{2}}{\sqrt{2}}\right)\right]$$

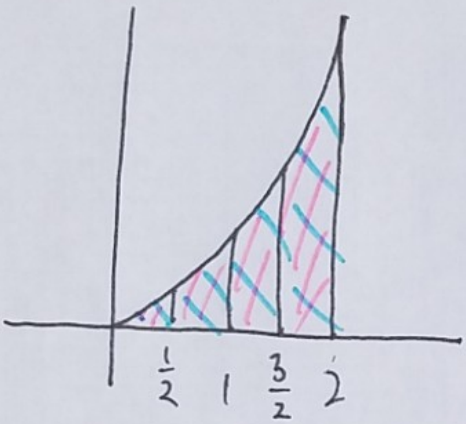
$$\boxed{= \frac{\pi}{4}\left(\frac{2+\sqrt{2}}{\sqrt{2}}\right)}$$

B.)
$$\int_0^{\pi} \sin x \, dx = \frac{\pi}{16}\left[\sin(0) + 2\sin\left(\frac{\pi}{8}\right) + 2\sin\left(\frac{\pi}{4}\right) + 2\sin\left(\frac{3\pi}{8}\right) + 2\sin\frac{\pi}{2} + 2\sin\left(\frac{5\pi}{8}\right) + 2\sin\left(\frac{3\pi}{4}\right) + 2\sin\left(\frac{7\pi}{8}\right) + \sin\pi\right]$$

$$= \frac{\pi}{16}\left[0 + 0.7654 + \frac{2}{\sqrt{2}} + 1.8478 + 2 + 1.8478 + \frac{2}{\sqrt{2}} + 0.7654 + 0\right]$$

$$= \frac{\pi}{16}\left[10.05467898\right] \approx 1.974$$

Ex: $\displaystyle\int_0^2 x^2\,dx$ $n=4$ $\frac{2-0}{2(4)} = \frac{2}{8} = \frac{1}{4}$



$= \frac{1}{4}\left[0^2 + 2\left(\frac{1}{2}\right)^2 + 2(1)^2 + 2\left(\frac{3}{2}\right)^2 + 2^2\right]$

$= \frac{1}{4}\left[0 + \frac{1}{2} + 2 + \frac{9}{2} + 4\right]$

$= \frac{1}{8} + \frac{1}{2} + \frac{9}{8} + 1$

$= \frac{1}{8} + \frac{4}{8} + \frac{9}{8} + \frac{8}{8}$

$= \frac{22}{8} = \frac{11}{4}$