

GEOMETRIC



$$\sum_{n=1}^{\infty} ar^n$$

CONVERGES
if $|r| < 1$

DIVERGES
if $|r| > 1$

Geometric grows by a set value
- rate is constant

$$\frac{\text{SUM}}{S = \frac{a}{1-r}}$$

Ex). $\sum_{n=1}^{\infty} 2\left(\frac{1}{2}\right)^n$ $|r| < 1$
 $|1/2| < 1 - \text{CONVERGES}$

$$S = \frac{2}{1-\frac{1}{2}} \quad S = \frac{\frac{2}{1}}{\frac{1}{2}} \quad S = 4$$

SUM = 4

Ex). $\sum_{n=1}^{\infty} (2)^n$ $|r| > 1$
 $|2| > 1 - \text{DIVERGES}$ (no sum)

Limit Comparison

$$\sum_{n=1}^{\infty} a_n \quad \sum_{n=1}^{\infty} b_n$$

$\lim_{n \rightarrow \infty} \frac{b_n}{a_n}$ ~~exists~~

If limit diverges, Both Series diverge.

If limit converges, both converge

Ex) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3 - 1}$ $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3}$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\frac{n^3 - 1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n^3} \cdot \frac{n^3 - 1}{\sqrt{n}} = \lim_{n \rightarrow \infty} \frac{n^3 - 1}{n^3} = 1 \text{ Converges.}$$

Sequences

Arithmetic: $a_n = a_1 + d(n-1)$ Next = Now + Rate

Geometric: $a_n = a_1(r)^{n-1}$ Next = Now • Rate

$\lim_{n \rightarrow \infty} a_n$ — When finding convergence of a sequence, you find the limit.

$$\lim_{n \rightarrow \infty} \frac{1}{n} = \left[1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n} \right] \begin{matrix} \text{approaches } 0 \\ \text{converges} \end{matrix}$$

$$\lim_{n \rightarrow \infty} n = \left[1, 2, 3, 4, \dots, n \right] \begin{matrix} \text{approaches } \infty \\ \text{diverges} \end{matrix}$$

If the limit approaches a finite number, it ~~diverges~~ converges.
If it approaches infinity, it diverges.

Integral

• $f \Rightarrow +$, continuous, and decreasing for
 $x \geq 1$ and $a_n = f(n)$

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x) dx$$

Both Converge or Both Diverge

Ex) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

$u = x^2 + 1$
 $\frac{1}{2} du = x dx$

$$\lim_{a \rightarrow \infty} \int_1^a \frac{x}{x^2+1} dx$$
$$\lim_{a \rightarrow \infty} \frac{1}{2} \int_1^a \frac{1}{u} du = \lim_{a \rightarrow \infty} \frac{1}{2} \ln|x^2+1| + c \Big|_1^a$$
$$\lim_{a \rightarrow \infty} \frac{1}{2} \ln|a^2+1| - \frac{1}{2} \ln|2| \quad \boxed{\text{DIVERGES}}$$
$$\infty - \frac{1}{2} \ln|2| \rightarrow \infty$$

Root

given $\sum_{n=1}^{\infty} a_n$, find $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L$

- ① if $L < 1$ series converges
- ② if $L > 1$ series diverges
- ③ if $L = 1$ root test doesn't work

* a_n must be positive *

* good for n^{th} powers *

Ex.

$$\sum_{n=0}^{\infty} \frac{3^n}{(n+1)^n} = \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{3}{n+1}\right)^n}$$

$$\lim_{n \rightarrow \infty} \frac{3}{n+1} = \emptyset$$

$L < 1$ so series converges.

Alt series

Ratio

(x-2)

Ratio

$$\text{find } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$$

① if $L < 1$ series converges absolutely

② if $L > 1$ series diverges

③ if $L = 1$ ratio test doesn't work

* helpful when $n!$ *

$$\text{Ex. } \sum_{n=1}^{\infty} \frac{n^2}{e^n} = \lim_{n \rightarrow \infty} \left| \frac{\frac{(n+1)^2}{e^{n+1}}}{\frac{n^2}{e^n}} \right| = \left(\frac{(n+1)^2}{e^{n+1}} \right) \left(\frac{e^n}{n^2} \right)$$

$$\left(\frac{(n+1)^2}{e^{n+1}(e)} \right) \left(\frac{e^n}{n^2} \right) \quad \lim_{n \rightarrow \infty} \frac{(n+1)^2}{(n^2)(e)} = \frac{1}{e} \quad \begin{matrix} \text{same degree} \\ \uparrow \\ L \end{matrix}$$

Since $L < 1$, series converges.

TELESCOPING

Given

$$\sum_{n=1}^{\infty}$$

a_n , look for fractions that

add to 0 when you expand the series.

(can be written out or turned into partial fractions)

Example:

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) \rightarrow \left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) \dots$$



Converges $1 \leftarrow 1 - 0 \leftarrow 1 - \lim_{n \rightarrow \infty} \frac{1}{n}$

~~THIS~~

~~CANCEL~~

P-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

- p must be positive
- CONVERGES if $p > 1$
- DIVERGES if $p \leq 1$ and $p > 0$

ex. $\sum_{n=1}^{\infty} \frac{1}{n} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}} \quad \frac{1}{2} < 1 \text{ so DIVERGES}$

ex. $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{4}{3}}} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{3}}} \Rightarrow 1 + \frac{1}{3} = \frac{4}{3} \quad \frac{4}{3} > 1 \text{ so CONVERGES}$

ex. $\sum_{n=1}^{\infty} \frac{1}{n} \quad 1 = 1 \text{ so DIVERGES}$

Alt. Series

$$\sum_{n=1}^{\infty} (-1)^n a_n \quad \text{or} \quad \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

* let $a_n > 0$

Converge if:

① $\lim_{n \rightarrow \infty} a_n = 0$

② $a_{n+1} \leq a_n$ for $\forall n$

Ex) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n}\right) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$

① $\lim_{n \rightarrow \infty} \frac{1}{n} = 0 \quad \checkmark$

② $\frac{1}{m+1} \leq \frac{1}{n}$ \checkmark

Converges by
Alt. Series test

Yours

Direct Comparison

let $0 < a_n < b_n$

think:
 $AD < BC$

1. if $\sum b_n$ converges, then $\sum a_n$ converges

2. if $\sum a_n$ diverges, then $\sum b_n$ diverges

Ex: $\sum_{n=1}^{\infty} \frac{1}{n^4+1}$ compare to $\sum_{n=1}^{\infty} \frac{1}{n^4}$ (make it simpler!)

• find out which one is bigger or smaller

$\frac{1}{n^4+1}$ is smaller (b/c the denominator is bigger)

• p-series: $\sum_{n=1}^{\infty} \frac{1}{n^4}$ $p=4 > 1 \rightarrow$ converges

so both are convergent

Ex: $\sum_{n=1}^{\infty} \frac{4^n}{7^n+3}$ compare to $\sum_{n=1}^{\infty} \frac{4^n}{7^n}$

• geometric test: $\sum_{n=1}^{\infty} \frac{4^n}{7^n}$

$$\frac{4^n}{7^n} = \left(\frac{4}{7}\right)^n \quad r = \frac{4}{7} < 1$$

↓
convergent

so both
are convergent

"Nth Term"

Method of determining divergence of a series

$$\sum_{n=1}^{\infty} a_n$$

Diverges if $\lim_{n \rightarrow \infty} a_n \neq 0$

Only for Divergence

Ex: $\sum_{n=1}^{\infty} \frac{n-1}{3n-1}$

$$\lim_{n \rightarrow \infty} \frac{n-1}{3n-1} = \frac{1}{3}$$

$\frac{1}{3} \neq 0$, therefore this series is divergent